Contents

1	Mathematical tools			
	1.1	Vector algebra	1	
	1.2	Differential operators on scalar and vector fields	3	
	1.3	Integration of vector fields. Gauss and Stokes theorems	5	
	1.4	Dirac delta	7	
	1.5	Fourier transform	13	
	1.6	Tensors and rotations	16	
	1.7	Groups and representations	19	
		1.7.1 Reducible and irreducible representations	20	
		1.7.2 The rotation group and its irreducible tensor representations	22	
2	Syst	cems of units	27	
	2.1	The SI system	27	
	2.2	Gaussian units	31	
	2.3	SI or Gaussian?	36	
3	Max	cwell's equations	39	
	3.1	Maxwell's equations in vector form	39	
		3.1.1 Local form of Maxwell's equations	39	
		3.1.2 Integrated form of Maxwell's equations	42	
	3.2	Conservation laws	43	
		3.2.1 Conservation of the electric charge	43	
		3.2.2 Energy, momentum and angular momentum of the		
		electromagnetic field	45	
	3.3	Gauge potentials and gauge invariance	52	
	3.4	Symmetries of Maxwell's equations	55	
		nentary applications of Maxwell's equations	57	
	4.1	Electrostatics	57	
		4.1.1 Coulomb's law	58	
		4.1.2 Electric field from a generic static charge density	59	
		4.1.3 Scalar gauge potential and electrostatic potential	62	
		4.1.4 Instability of a system of static charges	63	
		4.1.5 Uniqueness of the solution of electrostatic problems	64	
		4.1.6 Electrostatics of conductors	66	
		4.1.7 Electrostatic forces from surface integrals	68	
	4.2 Magnetostatics		71	
		4.2.1 Magnetic field of an infinite straight wire	71	

viii Contents

		4.2.2 Magnetic field of a static current density	76
		4.2.3 Force of a magnetic field on a wire and between	78
		two parallel wires4.2.4 Force between generic static current distributions	80
		4.2.5 Magnetic forces from surface integrals	82
	4.3	Electromagnetic induction	83
	1.0	4.3.1 Time-varying magnetic field and Lenz's law	83
		4.3.2 Induction on moving loops	84
	4.4	Solved problems	86
5	Elec	ctromagnetic energy	99
	5.1	Work and energy in electrostatics	99
	5.2	Energy stored in a static electric field	101
		5.2.1 Continuous charge distribution	101
		5.2.2 The point-like limit and particle self-energies	104
		5.2.3 Energy of charges in an external electric field	107
		5.2.4 Energy of a system of conductors	108
	5.3	Work and energy in magnetostatics	109
	5.4	Energy stored in a static magnetic field	111
	5.5	Forces and mechanical potentials	114
		5.5.1 Mechanical potentials for conductors	118
	F 0	5.5.2 Mechanical potentials in magnetostatics	124
	5.6	Solved problems	128
6		tipole expansion for static fields	133
	6.1	1	133
	6.2	Magnetic multipoles	139
		Pointlike electric or magnetic dipoles	141
	6.4	Multipole expansion of interaction potentials	144
		6.4.1 Electric multipoles in external field	144
		6.4.2 Interaction between the electric multipoles of two	147
		charge distributions6.4.3 Interactions of magnetic multipoles	147 150
	6.5	Solved problems	$150 \\ 152$
			102
7		cial Relativity	155
	7.1	The postulates	155
	7.2	Space and time in Special Relativity	159
		7.2.1 Lorentz transformations	159
		7.2.2 Causality and simultaneity	161
		7.2.3 Proper time and time dilatation	163
	7.9	7.2.4 Lorentz contraction	165
	7.3	The mathematics of the Lorentz group	166 166
		7.3.1 Four-vectors and Lorentz tensors	166
		7.3.2 Contravariant and covariant quantities7.3.3 Invariant tensors of the Lorentz group	168 170
		7.3.3 Invariant tensors of the Lorentz group7.3.4 Infinitesimal Lorentz transformations	$170 \\ 172$
		7.3.5 Decomposition of a Lorentz transformations	
		1.5.5 Decomposition of a Lorentz tensor under fotations	112

Contents ix

7.3.6 Covariant transformations of fields	173
7.3.7 More general lessons	177
7.4 Relativistic particle kinematics	177
7.4.1 Covariant description of particle trajectories	es 177
7.4.2 Action of a free relativistic particle	179
7.4.3 Relativistic energy and momentum, four-m	omentum 180
8 Covariant formulation of electrodynamics	183
8.1 The four-vector current	183
8.2 The four-vector potential A_{μ} and the $F_{\mu\nu}$ tensor	185
8.3 Covariant form of Maxwell's equations	186
8.4 Energy-momentum tensor of the electromagnetic :	field 188
8.5 Lorentz transformations of the electric and magne	etic fields 190
8.6 Relativistic formulation of the particle-field intera	
8.6.1 Covariant form of the Lorentz force equati	
8.6.2 The interaction action of a point particle	193
8.7 The field-theoretical approach to classical electrod	*
8.7.1 Euler-Lagrange equations of relativistic fie	
8.7.2 Lagrangian of the electromagnetic field	202
8.7.3 Noether's theorem	205
8.8 Solved problems	211
9 Electromagnetic waves in vacuum	217
9.1 Wave equations	217
9.2 Electromagnetic waves in the Lorenz gauge	219
9.3 Electromagnetic waves in the Coulomb gauge	222
9.4 Solutions for \mathbf{E} and \mathbf{B}	224
9.5 Polarization of light	228
9.6 Doppler effect and light aberration	229
10 Electromagnetic field of moving charges	235
10.1 Advanced and retarded Green's function	235
10.2 The Liénard-Wiechert potentials	242
10.3 Fields of charge in uniform motion	245
10.4 Radiation field from accelerated charges	249
10.5 Radiation from non-relativistic charges. Larmor fo	
10.6 Power radiated by relativistic sources	256
10.6.1 Relativistic generalization of Larmor's form	
10.6.2 Acceleration parallel to the velocity	260
10.6.3 Acceleration perpendicular to the velocity	262
10.7 Solved problems	264
11 Radiation from localized sources	269
11.1 Far zone fields for generic velocities	269
11.1.1 Computation in the Lorenz gauge	269
11.1.2 Computation in the Coulomb gauge	273
11.1.3 Radiated power and spectral distribution	275
11.2 Low-velocity limit and multipole expansion of the tion field	e radia- 278

 \mathbf{x} Contents

	11.2.1 Electric dipole radiation	284
	11.2.2 Radiation from charge quadrupole and magnetic	;
	dipole	285
	3 Near zone, far zone and induction zone	290
1	I Solved problems	292
	st-Newtonian expansion and radiation reaction	295
	Expansion for small retardation	295
1	2 Dynamics to order $(v/c)^2$	298
	12.2.1 The gauge potentials to 1PN order	301
	12.2.2 Effective dynamics of a system of point charges	303
1	12.2.3 Reduction of order of the equations of motion	310
1	3 Self-force and radiation reaction	312
	12.3.1 Classical extended electron models vs. regulariza- tion schemes	313
	12.3.2 Self-energy and mass renormalization	$313 \\ 317$
	12.3.2 Sen-energy and mass renormalization 12.3.3 Radiation reaction at 1.5PN order	323
	12.3.4 The Abraham-Lorentz-Dirac equation	$323 \\ 328$
	12.3.4 The Abraham-Ebrentz-Dirac equation 12.3.5 Covariant derivation of mass renormalization and	
	radiation reaction	334
		001
	ctromagnetic fields in material media	345
	Maxwell's equations for macroscopic fields	346
	2 The macroscopic charge density: free and bound charges	
	3 The macroscopic current density	349
	Maxwell's equations in material media	351
	$\mathbf{\tilde{b}}$ Boundary conditions on $\mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}$	353
1	Constitutive relations	356
	13.6.1 Dielectrics	$\frac{356}{358}$
	13.6.2 Metals 13.6.3 Diamagnetic and paramagnetic materials	$350 \\ 359$
1	⁷ Energy conservation	360
	B Solved problems	361
1	Solved problems	001
	quency dependent response of materials	365
	General properties of $\sigma(\omega), \epsilon(\omega)$	365
	2 Causality constraints and Kramers-Kronig relations	367
	B The Drude-Lorentz model for $\epsilon(\omega)$	370
	The Drude model of conductivity The dielectric function of metals	374
1) The dielectric function of metals	377
	ctromagnetic waves in material media	381
	Electromagnetic waves in dielectrics	381
	2 Phase velocity and group velocity	384
1	3 Electromagnetic waves in metals	386
	15.3.1 Transverse EM waves	387
4	15.3.2 Longitudinal EM waves and plasma oscillations	391
1	Electromagnetic waves in waveguides	394 205
	15.4.1 Maxwell's equations in a waveguide	395

Contents xi

15.4.2 Boundary conditions	at the surface of conductors 396	
15.4.3 TE, TM and TEM m	odes 397	
16 Scattering of electromagnetic	radiation 401	
16.1 Scattering cross section	401	
16.2 Scattering on a free electron	403	
16.3 Scattering on a bound electro	on 406	
A Electrodynamics in Gaussian	units 411	
References		
Index		