Preface

This book evolved from the notes of a course that I teach at the University of Geneva, for undergraduate physics students. For many generations of physicists, including mine, the classic references for classical electrodynamics have been the textbook by Jackson and that by Landau and Lifschits.[1] The former is still much used, although more modern and excellent textbooks with a somewhat similar structure, such as Garg (2012) or Zangwill (2013), now exist, while the latter is by now rarely used, even as an auxiliary reference text for a course. Because of my field-theoretical background, as my notes were growing I realized that they were naturally drifting toward what looked to me as a modern version of Landau and Lifschits, and this stimulated me to expand them further into a book.

[1] In the text we will refer to the latest editions of these books, Jackson (1998) and Landau and Lifschits (1975). However, these books went through many editions: the first edition of Jackson appeared in 1962, while that of Landau and Lifschits even dates back to 1939.

While this book is meant as a modern introduction to classical electrodynamics, it is by no means intended as a first introduction to the subject. The reader is assumed to have already had a first course on electrodynamics, at a level covered for instance by Griffiths (2017). This also implies a different structure of the presentation. In a first course of electrodynamics, it is natural to take a ‘bottom-up’ approach, where one starts from experimental observations in the simple settings of electrostatics and magnetostatics, and then moves toward time-dependent phenomena and electromagnetic induction, which eventually leads to generalizing the equations governing electrostatics and magnetostatics into the synthesis provided by the full Maxwell’s equation. This approach is the natural one for a first introduction because, first of all, gives the correct historical perspective and shows how Maxwell’s equations emerged from the unification of a large body of observations; furthermore, it also allows one to start with more elementary mathematical tools, for the benefit of the student that meets some of them for the first time, while at the same time discovering all these new and fundamental physics concepts. The price that is paid is that the approach, following the historical developments, is sometimes heuristic, and the logic of the arguments and derivations is not always tight.

For this more advanced text, I have chosen instead a ‘top-down’ approach. Maxwell’s equations are introduced immediately (after an introductory chapter on mathematical tools) as the ‘definition’ of the theory, and their consequences are then systematically developed. This has the advantage of a better logical clarity. It will also allow us to always go into the ‘real story’, rather than presenting at first a simpler version, to be later improved.

An important aspect of our presentation is that we keep distinct the discussion of electrodynamics 'in vacuum' (i.e., the computation of the electromagnetic fields generated by localized sources, in the region outside the sources) from the study of Maxwell's equations inside materials. The study of the equations 'in vacuum' reveals the underlying fundamental structure of the theory, while classical electrodynamics in material media is basically a phenomenological theory. Mixing the two treatments, because of a formal similarity among the equations, can be conceptually confusing. Until Chapter 12 we will focus uniquely on vacuum electrodynamics, while from Chapter 13 we study electrodynamics in materials.[2]

[2]This approach is different from, e.g. that of Jackson, or Zangwill. It is instead the same followed by Garg, and especially by Landau and Lifschits, that even separated the subjects into two different books: "The Classical Theory of Fields", Landau and Lifschits (1975), for vacuum electrodynamics, and "Electrodynamics of Continuous Media", Landau and Lifschits (1984) for electrodynamics in materials.

Focusing first on vacuum electrodynamics allows us to bring out the two most important structural aspects of the theory at its fundamental level, namely gauge invariance and the fact that Special Relativity is hidden in the Maxwell's equations. We will introduce immediately and in full generality the gauge potentials, and work out most of the equations and derivations of vacuum electrodynamics in terms of them. From a modern field-theoretical perspective, we know that classical electrodynamics is the prototype of a gauge theory, and the notion of gauge fields and gauge invariance is central to all modern particle physics, as well as to condensed matter theory. Similarly, after having duly derived from Maxwell's equations the most elementary results of electrostatics and magnetostatics, as well as the notions of work and electromagnetic energy and the expansion in static multipoles, we move as fast as possible to Special Relativity, introducing the covariant formalism and showing how Maxwell's equations can be reformulated in a covariant form.[3] Having in our hands the gauge potentials and the covariant formalism, most of the subsequent derivations in Chapters 8–12 are performed in terms of them, with a clear advantage in technical and conceptual clarity.

[3]By comparison, Jackson introduces the gauge potentials in full generality for the first time only after about 220 pages and Zangwill after about 500 pages, and their introduction is in general presented simply as a trick for simplifying the equations. However, their role is much more fundamental, since they are the basic dynamical variables in a field-theoretical treatment (which also implies that they will become the basic variables also when one moves to a quantum treatment). As for Special Relativity, Jackson introduces it only after more than 500 pages, while Zangwill relegates it to Chapter 22, after 820 pages, and Garg to Chapter 24.

Even if this book was born from my notes for an undergraduate course, and is meant to be used for such a course, it has obviously grown well beyond the original scope, and some parts of it are quite advanced. More technical sections, or whole chapters that are more specialized, are clearly marked, so that the book can be used at different levels, from the undergraduate student, to the researcher that needs to check a textbook as a reference. Classical electrodynamics, for its richness and importance, is a subject to which one returns over and over during a scientist's career.

Finally, an important point, when writing a textbook of electrodynamics, is the choice of the system of units. In mechanics, the transformation between systems such as c.g.s. (centimeter-gram-second) and m.k.s. (meter-kilogram-second) is trivial, and just amounts to multiplicative factors. However, in electromagnetism there are further complications. This has led to two main systems of units for classical electrodynamics: the SI system, and the Gaussian system. As we will discuss in Chapter 2, the essential difference is that, for electromagnetism, the SI system beside the units of length, mass and time, introduces a fourth independent base unit of current, the ampere, while in the Gaussian system the unit of charge, and therefore of current, is derived from the three basic units of length, mass and time.

The SI system is the natural one for applications to the macroscopic world: currents are measured in amperes, voltages in volts, and so on. This makes the SI system the obvious choice for laboratory applications and in electrical engineering, and SI units are by now the almost universal standard for electrodynamics courses at the undergraduate level. The Gaussian system, on the other hand, has advantages in other contexts, and in particular leads to neater formulas when relativistic effects are important.[4]

This state of affairs has led to a rather peculiar situation. In general, undergraduate textbooks of classical electromagnetism always use SI units; in contrast, more advanced textbooks of classical electrodynamics are often split between SI and Gaussian units, and all textbooks on quantum electrodynamics and quantum field theory use Gaussian units. The difficulty of the choice is exemplified by the Jackson's textbook, that has been the 'bible' of classical electrodynamics for generations of physicists. The second edition (1975), as the first, used Gaussian units throughout. However, the third edition (1998) switched to SI units for the first 10 chapters, in recognition of the fact that almost all other undergraduate level textbooks used SI units; then, from Chapter 11 (Special Theory of Relativity) on, it goes back to Gaussian units, in recognition of the fact that they are more appropriate than SI units for relativistic phenomena.[5] Gaussian units are also the most common choice in quantum mechanics textbooks: when computing the energy level of the hydrogen atom, almost all textbooks use a Coulomb potential in Gaussian units, $-e^2/r$, rather than the SI expression $-e^2/(4\pi\epsilon_0 r)$.[6]

In this book we will use SI units, since this is nowadays the almost universal standard for an undergraduate textbook on classical electrodynamics. However, it is important to be familiar also with the Gaussian system, as a bridge toward graduate and more specialized courses. This is particularly important for the student that wishes to go into theoretical high-energy physics where, eventually, only the Gaussian system will be used. We will then discuss in Section 2.2 how to quickly translate from SI to Gaussian units, and, in Appendix A, we will provide an explicit translation in Gaussian units of the most important results and formulas of the main text.

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[4]Actually, its real virtues appear when combining electromagnetism with quantum mechanics. In this case, the reduction from four to three base units obtained with the Gaussian system can be pushed further, using a system of units where one also sets $\hbar = c = 1$, with the result that one remains with a single base unit, typically taken to be the mass unit. In quantum field theory this system is so convenient that units $\hbar = c = 1$ are called *natural units* (we will briefly mention them in Section 2.2). As a consequence, all generalizations from classical electrodynamics to quantum electrodynamics (and its extensions such as the Standard Model of electroweak and strong interactions) are nowadays uniquely discussed using the Gaussian system (or, rather, a variant of it, Heaviside–Lorentz or rationalized Gaussian units, differing just by the placing of some 4π factors, that we will also introduce in Chapter 2), supplemented by units $\hbar = c = 1$.

[5]Among the other 'old-time' classics, Landau and Lifschits (1975) used Gaussian units, while the Feynman Lectures on Physics, Feynman *et al.* (1964), used SI. The first two editions of the classic textbook by Purcell used Gaussian units, but switched to SI for the 3rd edition, Purcell and Morin (2013). Among more recent books, SI is used in Griffiths (2017), Zangwill (2013) and Tong (2015), while Gaussian units are used in Garg (2012) (with frequent translations to SI units).

[6]The most notable exception is the quantum mechanics textbook Griffiths (2004), that uses SI units, consistently with the classical electrodynamics book by the same author.