

# Cosmology II

## Problem sheet 2 Thermal history I

### Exercise 1:

For the Robertson-Walker metric compute the scale factor  $a$  as a function of *conformal* time  $\eta$  for

(a) a radiation dominated, (b) a matter dominated, and (c) a  $\Lambda$  dominated universe.

### Exercise 2:

(a) Compute the pressure  $p$  of a gas of particles.

hint: Consider the change of momentum per time in a gas of particles.

(b) Using the relation derived in (a) show that

$$\frac{\partial p}{\partial T} = \frac{\rho + p}{T}. \quad (1)$$

### Exercise 3:

Derive the number of relativistic degrees of freedom  $g_*$  (defined in the lecture) above the electroweak scale  $T > 300$  GeV by counting all standard model particles.

### Exercise 4:

(a) How many neutrinos are contained in a volume of  $1 \text{ cm}^3$  today? (How does this compare to photons?)

(b) Assume that one neutrino is heavy,  $1 \text{ keV} > m_\nu \geq 10^{-2} \text{ eV}$ . Show that

$$\Omega_\nu = \frac{m_\nu}{94 h^2 \text{ eV}}. \quad (2)$$

hint:  $\zeta(3) = 1.20$