A new dynamical estimator of Ω

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Abstract. The streaming velocity $v_{12}(r)$, i.e., the mean relative velocity of pairs of galaxies at fixed separation r, measured from the redshift space galaxy correlation function was used in the past as a dynamical estimator of the effective cosmological density parameter [8, 13, 17]. Here we propose a new technique: measuring v_{12} directly from redshift-distance surveys. We present a simple closed-form expression, relating $v_{12}(r)$ to the two-point correlation function of mass density fluctuations, $\xi(r)$. Our formula accurately reproduces results of N-body simulations in a wide dynamical range. We also show how the v_{12} signal can be extracted from redshift-distance surveys, and how such observations can be used to estimate $\Omega^{0.6}{\sigma_8}^2$, where Ω is the cosmological density parameter and σ_8 is the standard normalization for rms mass density fluctuations. Combined with other observational constraints on $\beta \equiv \Omega^{0.6}\sigma_8$, such measurements can be used to break the degeneracy between Ω and σ_8 and each of these parameters can be estimated separately. This conference contribution is a terse summary of our two recently submitted papers: ref. [7] and [10].

1 An analytical model for $v_{12}(r)$

Most dynamical estimates of the cosmological density parameter, Ω, use the gravitational effect of departures from a strictly homogeneous distribution on objects such as stars and galaxies considered as test particles. One such dynamical estimator can be constructed by using an equation expressing the conservation of particle pairs in a self-gravitating gas. This equation was derived by Davis and Peebles [3, 13] from the BBGKY theory more than two decades ago, and since then it has successfully resisted theorists' attempts to find a closed form solution. Here we propose to apply the weakly nonlinear gravitational instability theory and the strongly nonlinear stable clustering solution as limiting cases to construct an approximate solution of the pair conservation equation. Our approximate solution is given by [10]

$$v_{12}(x,a) \;=\; -\tfrac{2}{3}\,Hrf\,\bar{\bar{\xi}}(x,a)\,\Big[\,1\,+\,\alpha\bar{\bar{\xi}}(x,a)\,\Big]\;, \tag{1}$$

where $v_{12}(x,a)$ is the magnitude of the mean (pair-weighted) relative velocity, $v_{12}(x,t)\,\vec{x}\,/x$, of a pair of particles at a comoving separation vector $\vec{x}$; a is the expansion factor, $r = ax$ is the proper separation, $H(a)$ is the Hubble parameter, while $\bar{\bar{\xi}}(x,a) \equiv \bar{\xi}(x,a)/[1+\xi(x,a)]$, and $\bar{\xi}$ is the two-point correlation funcion of matter density fluctuations, ξ, averaged over a ball of comoving radius x : $\bar{\xi}(x,a) = 3x^{-3}\int_0^x \xi(y,a)y^2\mathrm{d}y$. At the present cosmological time $a = 1$, $x = r$ and $H = 100\ h^{-1}\mathrm{km\ s^{-1}Mpc^{-1}}$. The function f is the usual logarithmic derivative of the linear growing mode solution, $D(a)$, $f \equiv d\ln D/d\ln a$ (see, e.g. §11 in LSS). For models with a vanishing cosmological constant ($\Lambda = 0$), and for zero curvature models with $\Lambda \neq 0$, $f \simeq \Omega^{0.6}$ (e.g. [14]). The parameter α is defined by

$$\alpha \;=\; \bar{\xi}^{(2)}(x,a)/\bar{\xi}^{(1)}(x,a)^2 \ , \tag{2}$$

where $\bar{\xi}^{(1)}$ and $\bar{\xi}^{(2)} = O(\bar{\xi}^{(1)})^2$ are the first two terms in the perturbative expansion for $\bar{\xi}(x,a)$. The general technique for deriving $\xi^{(2)}$ for density fluctuations with Gaussian initial conditions is well known [12, 15]. The parameter α depends on the logarithmic slope of the correlation function, $\gamma(x) \equiv -d\ln\xi^{(1)}(x,a)/d\ln x$. For a pure power-law ξ with γ in the range from 0 to 2, $\bar{\xi}^{(2)}$ can be expressed in terms of Euler's gamma fuctions [15]; for $0 < \gamma < 1.99$ these results are well approximated by

$$\alpha \;=\; 1.843 \;-\; 1.1\gamma \;-\; 8.2\times 10^{-4}\gamma^{10} \ . \tag{3}$$

Our approximate solution of the pair conservation equation is designed to bridge weakly nonlinear perturbation theory, valid for large separations and $|\xi| < 1$, with the stable clustering regime, valid for small separations and $\xi \gg 1$. For $\xi \to 0$, eq. (1) agrees exactly with the perturbative solution of the pair conservation equation; while for $r \to 0$, it closely approximates the stable clustering solution, $v_{12}(r) = -Hr$.

Eq. (1) was tested against high-resolution AP^{3}M simulations of 256^3 dark matter particles in periodic boxes of comoving volume $(240\,h^{-1}\mathrm{Mpc})^3$, kindly provided to us by the Virgo collaboration [9]. We have compared our theoretical predictions for v_{12} with simulations with different CDM-like initial spectra of density fluctuations, corresponding to four different sets of values Ω, Λ, H and σ_8. The last parameter, used to normalize the initial spectrum, is the rms matter density contrast in a sphere with a radius of $8\,h^{-1}$Mpc. Our predictions, based on eq.(1) are in excellent agreement with $v_{12}(r)$ measurements for dark matter particles in the N-body experimets in the entire dynamical range probed by the simulations ($0.1 \lesssim \xi \lesssim 10^3$; see ref. [10]). Moreover, our results agree as well with a set of $v_{12}(r)$ curves obtained for simulated "galaxies" of two different luminosity classes in the simulations of Kaufmann et al. [11]. The galaxies in the simulation did not trace the mass distribution – their correlation functions were different from the dark matter $\xi(r)$, however, their mean pairwise motions were identical with the motions of dark matter particles. These results suggest that it is not unreasonable to assume that real galaxies also trace the dark matter velocity field well even if they do not trace the mass distribution itself; the above results also strongly disagree with the "linear bias" model. The linear bias theory predicts $v_{12} \propto \sigma_8 \Omega^{0.6}$ at large separations [8], at variance with our eq. (1), which at large separations gives $v_{12} \propto {\sigma_8}^2 \Omega^{0.6}$, a significant difference unless the galaxy distribution is unbiased, i.e. $\sigma_8 = 1$.

2 The estimator

Since we observe only the line-of-sight component of the peculiar velocity, $s_A = \vec{r}_A \cdot \vec{v}_A / r \equiv \hat{r}_A \cdot \vec{v}_A$ (where $A = 1, 2 \ldots$ enumerate galaxies and with positions $\vec{r}_A$ and velocities $\vec{v}_A$) rather than the full three-dimensional velocity $\vec{v}_A$, it is not possible to compute v_{12} directly. Instead, we propose to use the mean difference between radial velocities of a pair of galaxies, $\langle\, s_1 - s_2 \,\rangle \;=\; v_{12}\, \hat{r} \cdot (\hat{r}_1 + \hat{r}_2)/2$, where $\vec{r} = \vec{r}_1 - \vec{r}_2$. To estimate v_{12}, we use the simplest least squares techniques, which minimizes the quantity $\chi^2(r) \;=\; \sum_{A,B} \left[(s_A - s_B) - p_{AB}\, \tilde{v}_{12}(r)/2\,\right]^2$, where $p_{AB} \equiv \hat{r} \cdot (\hat{r}_A + \hat{r}_B)$ and the sum is over all pairs at fixed separation $r = |\vec{r}_A - \vec{r}_B|$. The condition $\partial\chi^2/\,\partial\tilde{v}_{12} = 0$ implies [7]

$$\tilde{v}_{12}(r) \;=\; \frac{2 \sum (s_A - s_B)\, p_{AB}}{\sum {p_{AB}}^2} \quad . \tag{4}$$

To assess how useful this statistic is in practice we have conducted a series of tests with mock catalogues, identifying possible sources of systematic errors in $\tilde{v}_{12}$, including Malmquist bias. We also found ways of reducing these errors; these techniques were successfully tested with mock surveys (for a more detailed description, see ref. [7]). Preliminary results, obtained from the Mark III data [18] show a strong signal, in agreement with our expectations, based on experiments with mock data (paper in preparation). We are also working on estimating v_{12} from the SFI data [2] and expect to publish the results in the near future.

3 Comparison with other measures of Ω

Let us calculate the expected streaming velocity at $10\,h^{-1}$Mpc. One can use the APM catalogue of galaxies [5] for an estimate of γ at $10\,h^{-1}$Mpc. The resulting slope is $\gamma = 1.75 \pm 0.1$. Substituting $\gamma = 1.75$ into eqs. (1) and (3), we get

$$v_{12}(10h^{-1}\ \mathrm{Mpc}) \;=\; -\,667\, {\sigma_8}^2 \Omega^{0.6}\, (1 - 0.18{\sigma_8}^2)\,/(1 + 0.38{\sigma_8}^2)\, \mathrm{km/s}\,. \tag{5}$$

The above relation shows that at $r = 10h^{-1}$ Mpc, v_{12} is almost entirely determined by the values of two parameters: σ_8 and Ω. It is only weakly dependent on γ. This dependence is induced by the $\alpha\,\bar{\bar{\xi}}$ term in eq. (1). However, for all realistic values of γ, α is a small number. The uncertainties in the observed γ lead to an error in eq. (5) of less than 10% for $\sigma_8 \leq 1$. The above equation illustrates two important properties of v_{12} as an estimator of the density parameter: (i) it's dependence on σ_8 and Ω alone, and its independence from other model parameters, and (ii) a scaling with σ_8 and Ω, which is significantly different from the usual proportionality to $\beta \equiv \sigma_8 \Omega^{0.6}$.

At large separations $v_{12} \propto \Omega^{0.6} {\sigma_8}^2$. This scaling differs from that of other estimators like the POTENT method [16], cluster abundances [1, 6], and the position of the acoustic peaks in the cosmic microwave background fluctuation power spectrum [4], removing an important degeneracy. The advantage of using v_{12} as an estimator over the acoustic peak method is model-independence. The advantage of the streaming velocity estimates over the POTENT technique is simplicity and direct relation to observations.

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