Microlensing modulation by binaries

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A new effect on microlensing by binary systems coming mainly from the quadrupole is analyzed: the time dependence of the quadrupole can lead to specific modulations of the amplification signal. We study especially binary system lenses in our galaxy. The modulation is observable if the rotation period of the system is smaller than the time over which the amplification is significant and if the impact parameter of the passing light ray is sufficiently close to the Einstein radius so that the amplification is large. Observations of this modulation can reveal important information on the quadrupole and thus on the gravitational radiation emitted by the lens.

The importance of the quadrupole of a binary system relies mainly in its connection to gravitational radiation via Einstein's famous quadrupole formula [1]. This formula is beautifully confirmed by Taylor's binary pulsar [2, 3], the indirect proof of the existence of gravitational waves, for which Hulse and Taylor have been awarded with the Nobel price in 1993. Direct detection of gravity waves will (hopefully) be realized in the next few years by the numerous experiments operating today.

The question whether gravitational waves from binary systems can be detected via their effects on the propagation of photons has been addressed repeatedly in the past (see [4] to [9]). These studies focus on the deflection angle and on the time delay caused by a gravity wave passing through the path of the photon; the effects of quadrupole variation on the deflection angle turn out to be of the order of 10^{-6} arcsec, at the limit of the nowadays astrometry's possibilities, and the conclusion seems to be that the other effects are still too faint to be detectable today.

Kopeikin et al. [8, 9] study the problem in full generality, determining the time delay, Δ, and deflection angle, α, caused by a localized mass distriution, **D**, acting as a deflector of light (see Fig. 1). In [8, 9], a multipole expansion for the energy momentum tensor of the source is used. The gravity wave amplitude is proportional to the second time derivative of the quadrupole while the quadrupole term of the scalar potential contains no time derivatives, it decays like $1/d^3$ as in electrostatics. The ratio between the gravity wave amplitude and the quadrupole contribution to the scalar potential is therefore of the order of $(v/c)^2 \sim 10^{-6}$, even for compact binaries.

In this *letter* we study the contributions of the dipole and the quadrupole to the scalar potential which is responsible for lensing. We are not looking at gravitational waves whose contribution is negligible, as mentioned above. Especially, we want to compute the contribution of the quadrupole to microlensing. Mass and angular momentum are conserved quantities, while the quadrupole is in general time dependent. It will therefore introduce a time-dependence in the microlensing signal

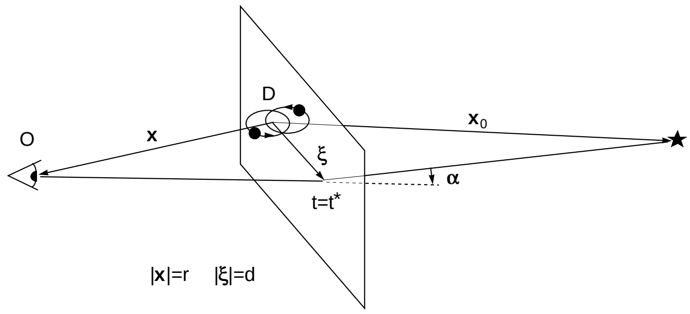

FIG. 1: The light from a source S is lensed by a deflector (D). The impact parameter ξ is a vector in the lens plane. The distances of D from the observer and from the light source respectively are r and r_0.

which we now determine. This effect is only visible if the period of the system is smaller than the time-scale of microlensing *i.e.* for relatively compact binaries with periods of less than about 30 days.

We work in the thin lens approximation, which means that we may project the lens mass distribution into a plane and we consider impact parameters $d = |\xi|$ much smaller than the distances r and r_0 in Figure 1. Furthermore, we assume the condition $\frac{\omega d^2}{cr} \ll 1$ where ω is the frequency of the binary, so that retardation inside the lens plane can be neglected. We employ the center of mass system, *i.e.* the coordinate system where the center of mass of the binary is at rest at position $\mathbf{x} = 0$. Up to the quadrupole, the gravitational lens potential is then given by [9]

$$\Psi(\xi, t^*) = \left[M + \epsilon_{jpq} k^p S^q \partial_j + \frac{1}{2} I^{TT}_{pq}(t^*) \partial_p \partial_q \right] \ln d \ , \quad (1)$$

where we have set $G = c = 1$ and t^* denotes retarded time, $t^* = t - r$. Here $\mathbf{k}$ is the unit vector pointing from the source to the observer, M is the total mass of the system, $\mathbf{S}$ is its angular momentum and I^{TT}_{pq} is the transverse traceless quadrupole tensor, projected into the plane normal to $\mathbf{k}$.

$$S^q(t) = \frac{1}{2}\epsilon^{qpr}\int d^3x\left(x^pT^{0r}(\mathbf{x},t) - x^rT^{0p}(\mathbf{x},t)\right) \quad (2)$$

$$I_{pq} = \int d^3x\rho(\mathbf{x},t)(x_qx_p - \frac{1}{3}|x|^2\delta_{qp}), \quad (3)$$

$$I^{TT}_{pq} = \Big[\delta_{ip}\delta_{jq} + \frac{1}{2}(\delta_{pq} + k_pk_q)k_ik_j - (\delta_{pi}k_qk_j + \delta_{qi}k_pk_j)\Big]I_{ij} \quad (4)$$

$T^{\mu\nu}(\mathbf{x})$ denotes the energy momentum tensor of the source and $\rho(\mathbf{x},t) = T^{00}$ is the energy density. Since the background spacetime is Minkowski, spatial index positions are irrelevant. The time delay and the deflection angle can be expressed in terms of Ψ as [9]

$$\Delta = -4\Psi + 2M\ln(4rr_0)\ , \qquad \alpha_i = 4\partial_i\Psi\ . \quad (5)$$

The amplification of a far away light source is $\mu = \frac{1}{\det(A)}$ where A is the Jacobian of the lens map (see e.g. [10]),

$$A_{ij} = \delta_{ij} - 4r\partial_i\partial_j\Psi(\xi, t^*). \quad (6)$$

Without loss of generality, we fix the orientation of the coordinate system so that the x_1 axis is aligned with the impact vector and the third axis is normal to the lens plane. Hence $\xi_1 = d$, $\xi_2 = 0$ and $k_1 = k_2 = 0$, $k_3 = 1$. From Eqs. (1,6) one then obtains the following expression for μ,

$$\mu^{-1} = \det(A) = 1 - \frac{16r^2}{d^2}\left[\frac{M^2}{d^2} + 4\frac{S_1^2 + S_2^2}{d^4} + 4M\left(\frac{S_2}{d^3} + \frac{3(I_{11} + \frac{1}{2}I_{33})}{d^4}\right) + 24\frac{S_2(I_{11} + \frac{1}{2}I_{33})}{d^5} - 24\frac{S_1I_{12}}{d^5} + \frac{36(I_{11} + \frac{1}{2}I_{33})^2}{d^6} + 36\frac{I_{12}^2}{d^6}\right]\ . \quad (7)$$

We use that in our coordinate system I^{TT}_{ij} is entirely determined by $I^{TT}_{11} = -I^{TT}_{22} = I_{11} + \frac{1}{2}I_{33}$ and $I^{TT}_{12} = I_{12}$. The quadrupole tensor I_{ij} has to be evaluated at retarded time $t^* = t - r$. The largest term containing the quadrupole is suppressed by a factor $\frac{|I_{11}|}{d^2M}$ with respect to the monopole term. If a denotes the mayor half axis of the binary, we have $|I_{ij}| \simeq Ma^2$, hence the suppression is of the order of $\epsilon^2 = (a/d)^2$. Therefore, one might suggest that systems with large orbits have the strongest contribution from the quadrupole. This is true, but then the time variation may not be visible if the period of the system is larger than the duration of the microlensing event, $T = 2\pi a^{3/2}/\sqrt{M} \geq d/v$. Here v is the source velocity (projected into the lens plane).

Furthermore, our approximation breaks down at $a \simeq d$ since higher multipoles can no longer be neglected and the microlensing event probes the full matter distribution of the lens. These microlensing modulations of binary lenses have been studied before, but mainly cases where the line of sight passes throught the caustics of the deflector system, so that $a < d$ is not satisfied, see Refs. [11] to [15]. In that case, all multipoles become important, and no information about the quadrupole can be gained. In this work we restrict ourselves to $a/d < 0.3$, say.

The amplification is largest close to the caustic line given by $\det(A) = 0$. Neglecting the sub-dominant contributions this corresponds to the Einstein radius $r_E = 2\sqrt{Mr} \equiv d_c$, the critical impact parameter. Close to the caustic, the effect of the quadrupole is strongly enhanced by a factor $\Delta^{-1} \equiv d_c/(d - d_c)$. To illustrate this, we consider a binary with angular momentum normal to the lens plane, and take into account only the dominant contribution from the quadrupole in Eq. (7), $\frac{M}{d}\frac{12I_{11}}{d^3}$. We parameterize this quadrupole term by $\frac{12(I_{11}+\frac{1}{2}I_{33})(t^*)}{d^3} = \gamma(t^*)\frac{M}{d}\epsilon^2$. Here $\gamma(t^*)$ is a dimensionless function of order unity. The amplification can then be approximated by

$$\mu^{-1} \simeq 1 - \left(\frac{d_c}{d}\right)^4\left[1 + \gamma\epsilon^2\right]\ , \quad \epsilon = \frac{a}{d}\ . \quad (8)$$

We want to consider the case $d = d_c(1 + \Delta)$ with $\Delta \ll 1$. In this case we have to lowest order in the small parameters ϵ and Δ

$$\mu \simeq \frac{1}{4\Delta - \gamma\epsilon^2} = \frac{\Delta^{-1}}{4 - \gamma\epsilon^2/\Delta}\ . \quad (9)$$

We conclude that the contribution from the quadrupole is significant if the ratio $\epsilon^2/\Delta \simeq 4\mu\epsilon^2$ is significant, say larger than a few percent.

As usual, our ray optical approach gives rise to a divergence of the amplification when the impact parameter approaches the critical value $d_c = 2\sqrt{M\cdot r}$. At distances smaller than d_c there are in principle multiple images, but they are too close together to be resolved by present optical telescopes. The divergences in the geometrical optics treatment is removed in the correct treatment using wave optics [10].

In Figs. 2 and 3 we present two examples of microlensing by binary systems of two equal masses $M_1 = M_2 = 1.4M_\odot$ in circular orbit in the lens plane so that the spin is aligned with the 3-axis, $S_1 = S_2 = 0$. We consider a background source moving with 100km/s relative to the lens $\mathbf{D}$.

In Fig. 2 the amplification is plotted as function of time for a neutron star or white dwarf binary. A rotation period of $T = 10^5$sec, corresponding to an orbital radius of $a \simeq 4.5 \times 10^6$km is assumed. The binary is placed at distance $r = 200$pc. The impact parameter is $d = (1 + 10^{-3})r_E$ yielding an amplification of about $\mu \simeq 250$. This is very close to the critical impact parameter $d_c = r_E = 2.2 \times 10^8$km. The quadrupole modulation amounts to 43% of the static contribution at maximum amplification. Our naïve estimate gives a relative contribution $4\mu\epsilon^2 \simeq 0.4$ from the quadrupole, which is in the

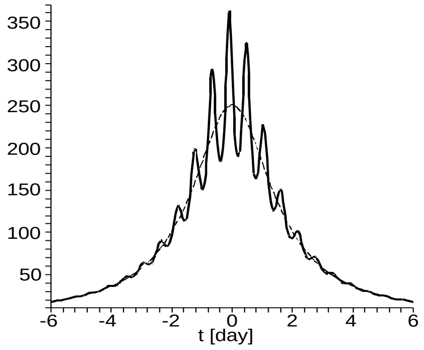

FIG. 2: The amplification is plotted as a function of time. The dashed line represents the mass (static) contribution and the solid line is the total (mass + quadrupole) signal. This corresponds to a microlensing event by a neutron star or white dwarf binary with parameters $T = 10^5 s = 1.2$day and $r = 200pc$. The impact parameter is $d = (1 + 10^{-3})d_c$.

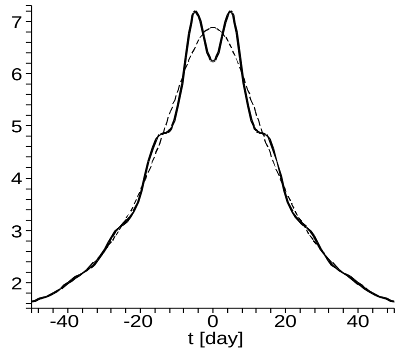

FIG. 3: As Fig. 2, but for a binary with parameters $T = 23$ days, $r = 700pc$, $d = (1.04)d_c$.

right ballpark.
In the second example, plotted in Fig. 3, we take a deflector somewhat further away, $r = 700$pc, and consider a binary with period, $T = 2 \times 10^6$sec = 23days. For this system, $a \simeq 3.4 \times 10^7$km. The impact parameter of this case is $d = 1.04 r_E$ yielding a maximal amplification of about 7. The quadrupole modulation amounts to 11% at maximum. Here the modulation signal is less significant since $\Delta = (d - d_c)/d_c$ is larger; also the period of the system is significantly longer.

The lines of equal amplification of a monopole lens are circles around the deflector. If a non-vanishing quadrupole moment is present, the circle of divergence as well as the circles of equal amplification are deformed as shown in Figure 4. They are simply the solutions of the equation $\mu^{-1} = 0$ and $\mu^{-1} =$ constant respectively, as a function of the angle between ξ and the direction of the vector relating the two stars of the binary. This vector, and with it the deformed circles, rotate with the period of the system. The observed modulation of the amplification comes from this rotation of the non-circular curves of constant amplification in the lens plane. We stress that the modulation can only be observed if the period of the binary is shorter than the time interval over which the amplification is significant.

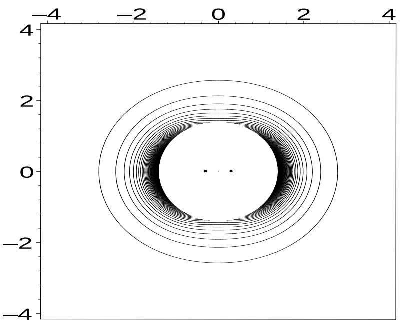

FIG. 4: Example of lines of equal amplification for a binary system. The dots indicate the position of the stars and the center of mass respectively.

If this effect is observed in a binary which emits gravitational waves in the LISA range of frequencies, this permits to determine the frequency and direction of the binary as source for gravitational waves. This will allow to detect the corresponding gravity wave signal out of the confusion noise in the LISA data [16].

The idea to detect gravity waves via microlensing has been recently studied in [17] and [18], but in these works the gravity wave source and the static lens are two different objects, the first being far enough from the second to make the quadrupole contribution to the scalar potential discussed here unimportant.
In this paper we consider the quadrupole variation of the deflector itself and we study its contribution to the scalar lens potential. The effect from the also emitted dynamical gravitational wave is much smaller than the one considered here in the frequency range we are interested in $(10^{-6} - 10^{-3} Hz)$, since it is proportional to the second time derivative of the quadrupole.
However, measuring the modulation of μ provides access to information about the variation of I_{ij} itself. As we have seen above, in order for the modulation to be measurable, the microlensing event has to reach rather high magnification.

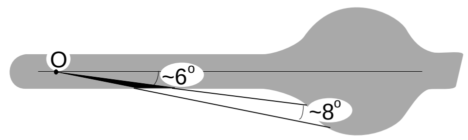

FIG. 5: Configuration for the bulge survey. The black triangle represents the volume used to determine N_b.

We conclude the *letter* with an estimate for the expected rate of such microlensing events. We focus on a galactic bulge survey where the observed field is about $8^o \times 8^o = (\Delta\varphi)^2$ centered on galactic coordinates $l \sim 4^o$, $b \sim -6^o$, which contains about $N_s = 5 \cdot 10^7$ bulge stars [19]. Using the binary population model of Tutukov–Yungelson [20] we can estimate the number of galactic black hole, neutron star and white dwarf binaries to be about 3×10^8. Within the volume swept by the light rays coming from our sources, we expect to find about a fraction (see Fig. 5)

$$x \sim \frac{r^3(\Delta\varphi)^2}{3D_{\rm gal}R_{\rm gal}^2\pi} \simeq 10^{-4}$$

of these binaries, leading to $N_b \simeq 27000$. Here $R_{\rm gal} \simeq 15$kpc is the radius of the galactic disk and $D_{\rm gal} \simeq 300$pc is its thickness and $r \sim D_{\rm gal}/2\frac{1}{\sin(6^o)} = 1.4$kpc is the apparent thickness of the galactic disc in the direction of the survey.
The cross section σ of the events for which the modulation is visible is about $\sigma = 0.6d_c^2$. With this the fraction f of the observed field covered by a binary per unit time is given $f = \sqrt{\sigma}v/A$, where v is the center of mass velocity of the source with respect to the binary and $A = r^2(\Delta\varphi)^2$ is the area of the observed field. With $r = 700$pc and $v = 100$km/sec we obtain an event rate

$$\eta = \frac{\sqrt{\sigma}vN_bN_s}{A} \sim 0.1 \text{ events/year} . \qquad (10)$$

Note also, that we have taken into account only compact binaries. Main sequence binaries which are sufficiently close, so that $a/d < 0.3$, say may very well contribute a more substantial event rate.

This event rate seems not out of reach of observations and it is already interesting to investigate whether such an event is not present in existing microlensing surveys, that is whether a varying quadrupole behavior may fit one of the exotic microlensing events detected so far.

Even if the modulation cannot be detected directly, the contributions from the quadrupole is sufficiently important that it has to be included in the error budget for microlensing events with high magnification, $\mu \geq 7$, if one wants to reach an accuracy in the predicted light curve of about 10%.

Let us finally determine also the spin contribution which is, as we shall see usually unimportant. The dominant term is $4MS/d^3$. Its relative contribution is of the order of $MSd^{-3}/(M^2d^{-2}) \simeq (a/d)a\omega = \epsilon v_o \ll 1$, where v_o denotes the orbital velocity of the binary, $v_o \simeq 10^{-3}\sqrt{\frac{M}{3M_\odot}\frac{10^7\text{km}}{a}}$. When d approaches d_c also this term is parametrically enhanced leading to a magnification

$$\mu_S \simeq \frac{\Delta^{-1}}{4(1+\epsilon v_o/\Delta)} . \qquad (11)$$

This term is significant only for very compact and therefore very fast binaries, $a \sim 10^3$km — 10^4km, which then have to be sufficiently close so that a/d is not too small. More precicely, one finds $\epsilon v_o = \sqrt{\frac{a}{2r}}$. Hence we need $\frac{a}{2r} \geq 10^{-2}/\mu^2$ for the spin amplification to amount to at least 10%. This term is also more difficult to disentangle from the monopole since it is time independent like the latter.

In this *letter* we have derived an new effect on microlensing by close binaries which leads to a modulation of the light curve. Its relative contribution being of the order of $4\mu(a/d)^2$, the effect is most significant for high magnification. Typical microlensing surveys towards the galactic bulge should detect about one such event every decade. But even in cases where it is not observed directly, the effect has to be included in the error budget for the microlensing light curve.

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