Cosmic Microwave Background anisotropies with mixed isocurvature perturbations

R. Trotta, A. Riazuelo and R. Durrer
Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH-1211 Genève 4, Switzerland
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In the light of the recent high quality data of the cosmic microwave background anisotropies, several estimations of cosmological parameters have been published. In this work we study to which extent these estimations depend on assumptions about the initial conditions of the cosmological perturbations, which are usually supposed to be adiabatic. We show that for more generic initial conditions, not only the best fit values are very different but the allowed parameter range enlarges dramatically. This raises the question which cosmological information (matter content of the Universe vs. physics of inflation) can be *reliably* extracted from these data.

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Introduction. The discovery of anisotropies in the cosmic microwave background (CMB) by the COBE satellite in 1992 [1] has stimulated an enormous activity in this field, which has culminated recently with the high precision data of the BOOMERanG [2], DASI [3] and MAXIMA-1 [4] experiments. The CMB is developing into the most important observational tool to study the early Universe. So far, these data have however mainly been used to *estimate* cosmological parameters for a specific model of initial fluctuations, namely scale invariant adiabatic perturbations [5–12]. In all presently known working models of cosmic structure formation, initial conditions come from an early inflationary phase of the universe. The simplest models of inflation do indeed lead to adiabatic perturbations. However, string cosmology models predict isocurvature perturbations or a mixture of isocurvature and adiabatic perturbations, where the isocurvature mode closely resembles the neutrino isocurvature density NID mode discussed below [13–15]. Also ordinary inflationary models with more than one scalar field generically predict mixtures of adiabatic and isocurvature fluctuations [16,17].

Apart from a stochastic background of gravity waves, CMB anisotropies are so far our only window to the physics of inflation and hence to the physics at strings or even Planck scale. It is therefore crucial that we learn as much as possible about the physical mechanisms of inflation from these data.

In this work we investigate to which extent the determination of cosmological parameters depends on assumptions about initial conditions. We show in a specific example how the allowed parameter range is enlarged when the usual requirement for purely adiabatic initial conditions is relaxed. In order to limit the computational effort, we have chosen to vary some cosmological parameters and keep the others fixed. We set the total density parameter $\Omega_{\rm tot} \equiv \Omega_\Lambda + \Omega_{\rm m} = 1$ and fixed $\Omega_{\rm m} \equiv \Omega_{\rm c} + \Omega_{\rm b} = 0.3$ and $\Omega_\Lambda = 0.7$, where $\Omega_{\rm c}$ and $\Omega_{\rm b}$ are the density parameters of cold dark matter (CDM) and baryons respectively, and Ω_Λ denotes the density parameter due to a cosmological constant, $\Omega_\Lambda \equiv \Lambda/3H_0^2$, and $H_0 \equiv 100h$ km s^{-1} Mpc^{-1} is the Hubble parameter today. For fixed Ω_Λ, $\Omega_{\rm m}$ and spectral index $n_{\rm S} = 1$, we determine the parameters h and $\omega_{\rm b} \equiv \Omega_{\rm b}h^2$ for generic (i.e. mixed adiabatic and isocurvature) initial conditions. We also comment the question: what is the preferred isocurvature contribution to the perturbations? We shall see that, with present CMB data, this question cannot be answered without strong assumptions about the cosmological parameters.

Initial conditions. Some observable consequences of deviations from a pure adiabatic model were first investigated in Ref. [18]. So far, only one study considering the adiabatic mode together with just one isocurvature mode has been undertaken recently [19]. To choose a more generic set of initial conditions we follow the procedure outlined in Ref. [20]. In our model, the matter components of the universe are CDM, baryons, massless neutrinos, and photons. Apart from the adiabatic mode, one can show that perturbations can have a baryon isocurvature mode (BI), a CDM isocurvature mode (CI), a neutrino isocurvature density mode (NID), and a neutrino isocurvature velocity mode (NIV), the precise definition of which is given in Ref. [20]. We have noticed that implementing the initial conditions for all the modes was simpler and numerically unproblematic in a gauge-invariant formalism as compared to synchronous gauge (see [21] for details). The most generic initial conditions for five modes are then given by a positive semi-definite 5×5 matrix M representing the amplitude of each of the modes, including all the possible cross-correlations.

For a fixed set of cosmological parameters, we first compute the CMB anisotropy spectrum C_ℓ^{ij} when only one of the elements of the correlation matrix is non-zero ($M_{ij} = 1$, all other elements vanish) with a fixed spectral index $n_{\rm S} = 1$ for all modes. We then set

$$C_\ell(M) = \sum_{i,j=1}^{5} M_{ij} C_\ell^{ij} \ . \tag{1}$$

As already noticed in Ref. [22], the BI and CI components of the correlation matrix are identical, up to a multiplicative constant. We have therefore restricted our analysis to the four modes AD, CI, NID, NIV without loss of generality. We vary the correlation matrix M and the cosmological parameters $\omega_{\rm b}$ and h to search for the best fit to the data using a maximum likelihood method.

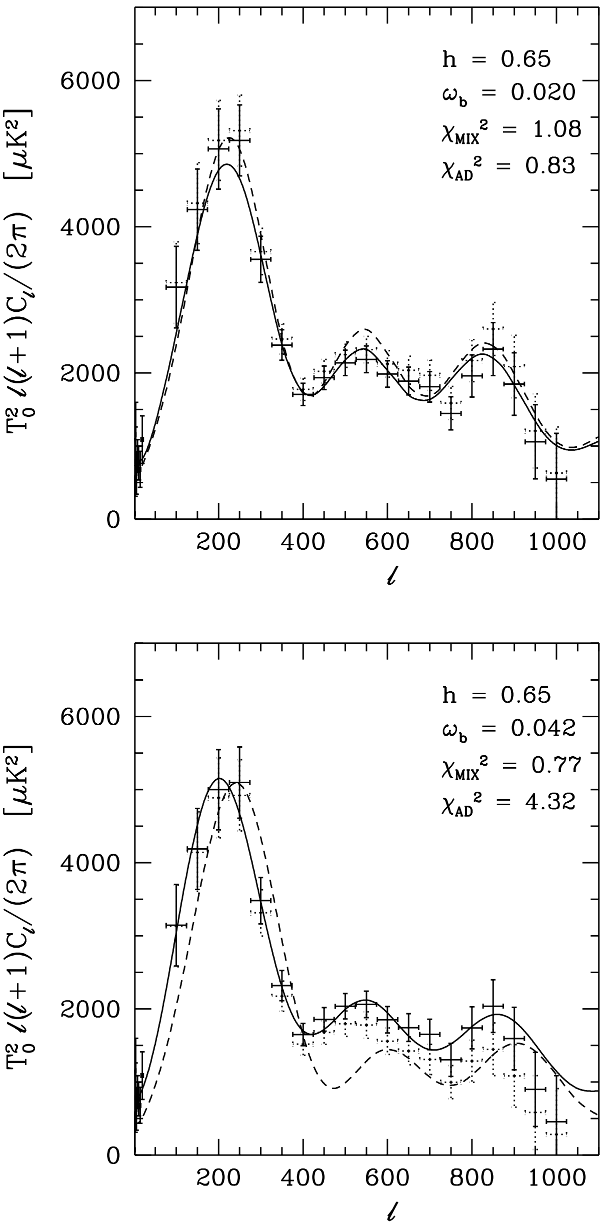

FIG. 1. CMB anisotropy spectrum for different values of the cosmological parameters $\omega_{\rm b}$ and h. We have shown the best-fit corresponding to a purely adiabatic case (dashed line) and allowing general initial conditions, mixed models (solid line). The calibration and the beam size of the BOOMERanG data have been optimized to fit the mixed model (solid error bars) or the adiabatic model (dotted error bars). The parameter choice on top ($\omega_{\rm b} = 0.02$, $h = 0.65$) can be fitted by both models while the values $\omega_{\rm b} = 0.042$, $h = 0.65$, can only be fitted by a mixed model.

Data analysis. We restrict our analysis to the COBE [1] and BOOMERanG [2] data. For the latter, we take into account the calibration and the beam size uncertainties [2] which we treat just like two additional (normally distributed) parameters of the problem. The fits are computed using a downhill simplex method [23] initiated after choosing a starting point randomly. The positive semi-definiteness of the correlation matrix M is ensured by penalty functions (more details are given in [21]). The best fit is then estimated after 15,000 minimization runs using this procedure. It turns out that the topology of the χ^2 surface on our 14-dimensional parameter space is quite complicated with many local minima and probably many degeneracies (see also the example discussed in [19]).

In Fig. 1 we show the best fit spectra for two different choices of the cosmological parameters $\omega_{\rm b}$ and h. Both of them are good fits if we allow for mixed initial conditions. On the plot we have also indicated the reduced χ^2. For a fixed choice of $\omega_{\rm b}$, h the purely adiabatic model has only 3 parameters (the amplitude of the adiabatic mode, the BOOMERanG calibration and beam size). With 26 data points (7 from COBE and 19 from BOOMERanG) this leads to $F_{\rm AD} = 26 - 3 = 23$ degrees of freedom. The mixed models have a symmetric 4×4 matrix determining the initial amplitude, leading to a total of 12 parameters and hence $F_{\rm MIX} = 14$ degrees of freedom. If we also vary $\omega_{\rm b}$ and h, the number of degrees of freedom is lowered by 2. It is not surprising that for fixed values $h = 0.65$, $\omega_{\rm b} = 0.02$, which are well fitted by the adiabatic model, the *reduced* χ^2 of the adiabatic model is somewhat lower than the one of the mixed model, since $F_{\rm MIX} < F_{\rm AD}$ (as an example, see top panel of Fig. 1). For the mixed model, the *absolute* χ^2 is always lower.

For both models we determine the likelihood functions of the cosmological parameters $\omega_{\rm b}$ and h by marginalizing over the initial conditions and the BOOMERanG calibration and beam size. The result is shown in Fig. 2 where the likelihood contours in the $(\omega_{\rm b}, h)$ plane for likelihoods of 50%, 68%, 95%, 99% are indicated for purely adiabatic models (top) and for mixed models (bottom). These plots represent the main result of our paper. It is remarkable to which extent the innermost good fit contour opens up, once we allow for isocurvature components. Strangely, the only excluded region which remains is the upper left corner contains the value of $\omega_{\rm b} = 0.019 \pm 0.02$ inferred from big bang nucleosynthesis (BBN) [24] and the Hubble space telescope key project value for the Hubble parameter [25] of $h = 0.72 \pm 0.08$. On the contrary, there is absolutely no upper limit for $\omega_{\rm b}$ within the regime investigated here! This is explained by the fact that the strongest features of a high baryon density universe, the asymmetry between even and odd acoustic peaks and the shift of the peak position due to the change in the sound velocity, can be fully compensated by an admixture of isocurvature modes (see lower panel of Fig. 1). A very high baryon density can therefore easily be accommodated in this framework. However, for high ω_b and low h, it is difficult to find a good fit because there is not enough power in the secondary peak region since the early integrated Sachs-Wolfe effect boosts the first peak.

χ^2 by variation of the initial conditions for given values of the cosmological parameters. Clearly, the further we move away from the parameter region well fitted by the purely adiabatic model, the higher becomes the isocurvature contribution needed to fit the data.

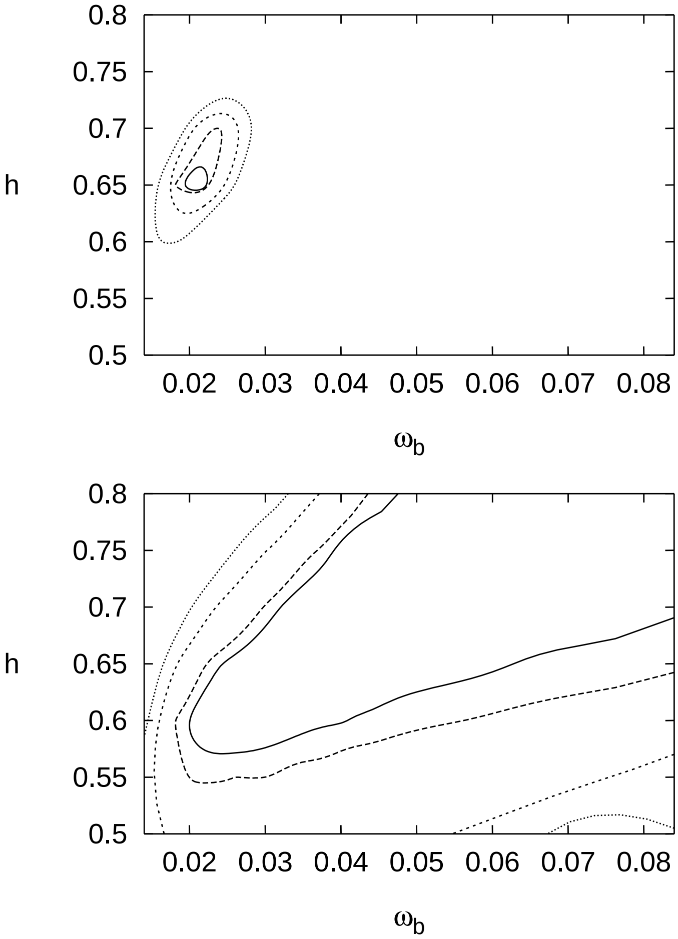

FIG. 2. The likelihood contours of 50%, 68%, 95%, 99% are indicated in the (ω_b, h) plane for purely adiabatic models (top) and for mixed models (bottom). The likelihoods are obtained by marginalization over the BOOMERanG calibration and beam size, as well as over the initial conditions given by the amplitude of the adiabatic mode for adiabatic models and by the matrix M for mixed models. For mixed models, the lowest χ^2 corresponds to even higher values of ω_b and h than those shown in the plot.

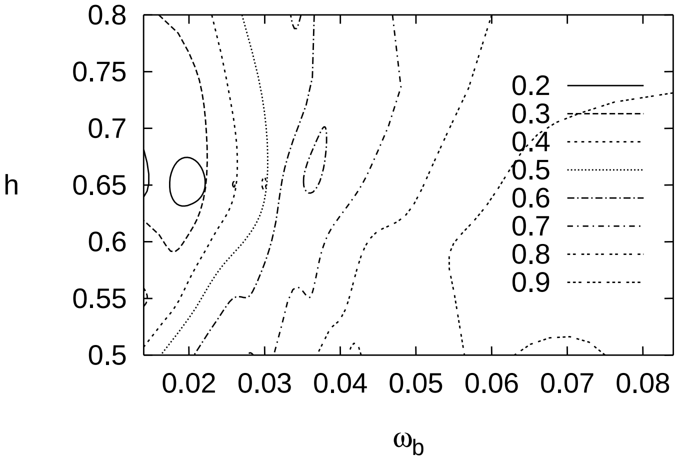

FIG. 3. The isocurvature content α of the best fit mixed model as function of the parameters (ω_b, h) is indicated. The contours $\alpha = 0.2$ to 0.9 in steps of 0.1 are shown.

We define the isocurvature content of a mixed model by $\alpha = (M_{22} + M_{33} + M_{44})/\mathrm{trace}M$, where M_{11} denotes the adiabatic mode amplitude. The isocurvature content in the model shown in the top panel of Fig. 1 is only $\alpha = 0.12$, while for the parameter choice in the bottom panel one has $\alpha = 0.69$. Hence, if the cosmological parameters are close to those chosen in the top panel, we can conclude that the cosmic perturbations are predominantly adiabatic. In Fig. 3 we show the isocurvature content α of the best fit model obtained by minimizing

The main non-adiabatic component of our best fits is the NID mode. This was to be expected, since this mode and its correlator with the adiabatic mode can shift the peak positions and can substantially add or subtract from the second peak [20]. A crucial point is therefore to know whether such a mode can appear in a realistic structure formation scenario. It is known that for interacting species the non adiabatic part of the perturbations tends to decay with time. Therefore, the generation of an NID component can only occur after neutrino decoupling, that is at $T \lesssim 1\,\mathrm{MeV}$. Whether or not such a phenomenon can occur at low energy is an open question. However, a neutrino isocurvature perturbation can also be due to a fourth species of sterile neutrinos which may have decoupled very early in the history of the Universe. The same remark applies of course also to the CDM isocurvature mode. Note that the energy density of this fourth neutrino type cannot be very high in order not to contradict the light element abundances, but there is nothing which prevents (at least in principle) the presence of large perturbations in this component.

Conclusion. We have shown that allowing for isocurvature perturbations, one may very well fit present CMB data with cosmological parameters which differ considerably from the ones preferred by adiabatic perturbations. More important, allowing for generic initial conditions, the ranges of cosmological parameters which can fit the CMB anisotropy data widen up to an extent to become nearly meaningless.

On the other hand, assuming measurements of cosmological parameters from other methods like direct measurements of the Hubble parameter which yield $h \sim 0.65$ and BBN which implies $\omega_{\rm b} \sim 0.02$, we can use the CMB to limit the isocurvature contribution in the initial conditions (or other unconventional features) and thereby *learn something about the very early universe*, i.e., the inflationary phase which has generated these initial conditions. *We can constrain viable models of inflation.*

For cosmological parameters in the range preferred by other, CMB independent, measurements ($\Omega_\Lambda \sim 0.7$, $\Omega_{\rm m} \sim 0.3$, $h \sim 0.65$, $\omega_{\rm b} \sim 0.02$) the isocurvature contribution in the initial conditions has to be relatively modest ($\alpha \lesssim 0.3$). Especially, we have checked that, given these cosmological parameters, *a purely isocurvature model, i.e. one with $M_{11} = 0$ cannot fit the data.*

Finally, and most importantly, our work shows the danger of calling parameter estimation by CMB anisotropy experiments a "parameter measurement" since the results depend so sensitively (and quite unexpectedly) on the underlying model assumptions. We rather consider CMB anisotropies as an excellent tool to test model assumptions or consistency. In the light of these findings, the importance of non-CMB measurements of cosmological parameters can clearly not be overstated. In short, CMB is the ideal tool to investigate the *primordial* parameters for cosmic structure formation (i.e., the initial conditions), while there are many other possibilities to constrain *cosmological* parameters (Ω's, h etc), which we have to use in order to obtain good limits for possible isocurvature perturbations.

As has been shown in Ref. [22], CMB anisotropies alone, even if measured with optimal precision limited by cosmic variance as proposed by the PLANCK experiment [26], do not allow to remove the degeneracy between cosmological parameters and initial conditions. Polarization measurements will represent an additional non-trivial mean to remove this degeneracy and might limit an isocurvature contribution to about 10%. In the same vein, using the normalization of the matter power spectrum (provided it can be measured accurately) also helps to break some of the degeneracies induced by the isocurvature modes.

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[1] G.F. Smoot *et al.*, Astrophys. J. **396**, L1 (1992); C.L. Bennett *et al.*, Astrophys. J. **430**, 423 (1994); M. Tegmark and A.J.S. Hamilton, in "18th Texas Symposium on relativistic astrophysics and cosmology", edited by A.V. Olinto *et al.*, pp 270 (World Scientific, Singapore, 1997).
[2] C.B. Netterfield *et al.*, preprint astro-ph/0104460 (2001).
[3] N.W. Halverson *et al.*, preprint astro-ph/0104489 (2001).
[4] A.T. Lee *et al.*, preprint astro-ph/0104459 (2001).
[5] A. Balbi, *et al.*, Astrophys. J. **545**, L5 (2000).
[6] A.H. Jaffe *et al.*, Phys. Rev. Lett. **86**, 3475 (2001).
[7] M. Tegmark and M. Zaldarriaga, Phys. Rev. Lett. **85**, 2240 (2000).
[8] M. Tegmark, M. Zaldarriaga and A. Hamilton, Phys. Rev. D **63**, 043007 (2001).
[9] A. Lange *et al.*, Phys. Rev. D **63**, 042001 (2001).
[10] C. Pryke *et al.*, preprint astro-ph/0104490 (2001).
[11] R. Stompor *et al.*, preprint astro-ph/0105062 (2001).
[12] P. de Bernardis *et al.*, preprint astro-ph/0105296 (2001).
[13] A. Melchiorri, F. Vernizzi, R. Durrer and G. Veneziano, Phys. Rev. Lett. **83**, 4464 (1999).
[14] K. Enqvist and M. Sloth, preprint hep-ph/0109214 (2001)
[15] D. Wands and D. Lyth, preprint hep-ph/0110002 (2001).
[16] A. Starobinsky, S. Tsujikawa and J. Yokoyama, Nucl.Phys. **B610** 383 (2001).
[17] D.H. Lyth and A. Riotto, Phys.Rept. **314** 1 (1999).
[18] D. Langlois and A. Riazuelo, Phys. Rev. D **62**, 043504 (2000).
[19] L. Amendola *et al.*, preprint astro-ph/0107089 (2001).
[20] M. Bucher, K. Moodley and N. Turok, Phys. Rev. D **62**, 083508 (2000).
[21] R. Trotta, Diploma thesis at ETH Zürich (2001).
[22] M. Bucher, K. Moodley and N. Turok, preprint astro-ph/0012141 (2000).
[23] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, *Numerical Recipes*, (Cambridge University Press, Cambridge, England, 1988).
[24] S. Burles, K.M. Nollet and M.S. Turner, Astrophys. J. **552**, L1 (2001).
[25] W.L. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).
[26] See webpage `http://astro.estec.esa.nl/PLANCK` .