Cosmological Structure Formation with Topological Defects

Abstract

A numerical investigation of cosmological structure formation is presented. Our model of structure formation is based on the idea that topological defects due to a phase transition in the early universe may have seeded the formation of structure. We describe the numerical simulation of the formation and evolution of topological defects in the universe. We stress especially the calculation of the resulting angular power spectrum of anisotropies in the cosmic microwave background radiation, which can be compared with present and future observations.

1 Introduction

In the 20th century the quest after the universe as a whole, cosmology, has become a quantitative science. The big bang model of a homogeneous and isotropic, adiabatically expanding universe has been very successful. It explains the cosmic expansion, predicts correctly the abundances of the light elements (H, ${}^2H$, ${}^3He$, ${}^4He$ and ${}^7Li$) and the existence of the cosmic microwave background radiation (CMB). This is the thermal bath of photons which have been scattered for the last time off the cosmic electrons just before the recombination of electrons and protons to neutral hydrogen. The isotropy of the CMB implies that on large enough scales the matter distribution is very isotropic and homogeneous.

However, the formation of cosmological structure on smaller scales i.e. inhomogeneities in the matter distribution like galaxies, clusters, super-clusters, voids and walls [1] is still an essentially unsolved problem.

At first sight it seems obvious that small density enhancements can grow by gravitational attraction. But global expansion of the universe and radiation pressure counteract gravity, so that, e.g., in the case of a radiation dominated, expanding universe no density inhomogeneities can grow faster than logarithmically. Even in a universe dominated by pressureless matter, dust, growth of density perturbations is strongly reduced due to the expansion of the universe.

On the other hand, we know that the universe was extremely homogeneous and isotropic at some early time. We conclude this from the isotropy of the 3K cosmic microwave background (CMB), which represents a relic of the baryon–electron–radiation plasma at times before protons and electrons recombined to hydrogen. Measurements with the DMR instrument aboard the NASA satellite "COsmic Background Explorer"

(COBE), have found anisotropies [2] on the level of

$$\frac{\Delta T}{T}(\theta) \sim 10^{-5} \quad \text{on angular scales } 7^o \leq \theta \leq 90^o \ .$$

On smaller angular scales the observational situation is somewhat unclear. Seemingly contradictory results exist in different regions of the sky. Many upper limits certainly require $\Delta T/T \lesssim 3.5 \times 10^{-5}$ on all scales $\theta < 8^o$. Some papers announce positive measurements in the range, $\Delta T/T = (1 \text{ to } 3) \times 10^{-5}$ on angular scales around $\theta \sim 1^o$ [3, 4].

The simplest model of a purely baryonic universe, $\Omega \sim 0.1$, with adiabatic initial fluctuations cannot explain this set of data. Either the initial perturbations are too large to satisfy CMB anisotropy limits, or they are too small to develop into the observed large scale structure.

The most conservative way out where one just assumes non adiabatic initial conditions (minimal isocurvature model) also faces severe difficulties [5, 6, 7, 8]. Other models assume that initial fluctuations were created during an inflationary epoch, but that the matter content in the universe is dominated by hot or cold dark matter or a mixture of both. Dark matter particles do not interact with photons other than gravitationally and thus induce perturbations in the CMB only via gravitation. In these models it is generally assumed that inflation leads to $\Omega = 1$, but $\Omega_B h^2 \sim 0.01$, which is compatible with the light element abundances. If one assumes only one component of dark matter, these models do not seem to convey with observations [9, 7]. But if a suitable mixture of hot and cold dark matter is adopted the results of numerical simulations look quite promising [10, 11, 12].

All these dark matter models assume that initial fluctuations are formed during an inflationary phase. Due to the conceptual and technical difficulties of present models of inflation (e.g., all of them have to invoke some amount of fine tuning to obtain the correct amplitude of density inhomogeneities), we consider it very important to investigate yet another possibility: Density perturbations in the dark matter and baryons might have been triggered by seeds. With seeds we mean an inhomogeneously distributed form of energy which makes up only a small fraction of the energy density of the universe.

Topological defects are especially natural types of seeds. They can form during symmetry breaking phase transitions in the early universe. Depending on the topology of the vacuum manifold of the cooler, less symmetric phase, 2–dimensional domain walls, 1–dimensional cosmic strings, point-like monopoles, unstable textures or a mixture of them all can form [13, 14]. If the symmetry is global, we speak of global defects, if it is gauged, the defects are called local. Such topological defects are well known in many physical systems. They are the vortices in Type II superconductors and the declinations in liquid crystals [15]. The formation and evolution of defects can be described by a scalar field which is called the order parameter in solid state physics or the Higgs field in particle physics.

Domain walls and local monopoles are disastrous for cosmology since they dominate the energy density of the universe soon after the phase transition. Local textures thin out. Global and local strings, global monopoles and global textures obey a so called scaling solution, i.e., they make up always about the same fraction of the energy density of the universe. This fraction which is of the order of GT_c^2 has to be about 10^{-6}. Here T_c is the phase transition temperature and G is Newton's gravitational constant. In units of GeV (Giga-electron Volts)[1] this yields a critical temperature of about $T_c \simeq 10^{16}$GeV, approximately the temperature where particle physics considerations predict the so called grand unified (GUT) phase transition.

During the last years gauge–invariant (i.e. invariant under linearized coordinate transformations) perturbation equations have been developed to treat cosmological perturbations in the presence of seeds[16, 17, 18, 7, 19].

These methods can be used to determine the anisotropies induced in the CMB and in the dark matter in the presence of topological defects. We have solved the equations exactly for the case of a spherically symmetric collapsing texture in flat space [16].

2 Numerical Simulations

With the help of numerical simulations we now want to test the hypothesis that the structure in the universe was seeded by global topological defects. The basic ingredients to for the simulations are the following:

We start at a time when the grid spacing of our fixed cubic lattice is approximately the size of the cosmic horizon. We then assume that the scalar field which describes the topological defects is uncorrelated at different lattice sites and we lay down the field randomly. Then we evolve it using the non–linear sigma model equations of motion. The scalar field undergoes a self ordering process on the scale of the horizon and its energy density soon enters a scaling regime where it decays like $1/t^2$ (see Fig. 1). This is numerically the hardest part since it involves the solution of 8 coupled non–linear partial differential equations. We then calculate the energy momentum tensor of the scalar field and use it as source term in Einstein's field equation which we solve in first order perturbation theory. The induced perturbations in the geometry lead to fluctuations in the distribution of dark matter and radiation which we also determine in first order perturbation theory. Finally, we compare the resulting angular power spectrum of CMB fluctuations and the 3d power spectrum of dark matter fluctuations with observations.

In our numerical simulations we have mainly concentrated on texture defects which are described by a four component scalar field, but we have also looked at global

[1] Setting $k_{Boltzmann} = 1$, we usually express temperatures in units of energy. 1 Electron volt (eV) is the kinetic energy gained by an electron traversing a tension of 1 Volt.
Setting furthermore Planck's constant, $\hbar$ and the speed of light, c equal 1, $\hbar = c = 1$, Newton's constant can be expressed in units of $(\text{energy})^{-2}$, $G = 0.67 \times 10^{-38} GeV^{-2}$.

monopoles and strings. After a spherically symmetric approximation [20, 21], we have performed a full 3 dimensional simulation of the texture field.

2.1 The angular power spectrum on large scales

On large angular scales, the CMB anisotropies are due to the fact that photons reaching us from different directions travel through slightly different geometries. Taking just this effect into account, we have determined the induced angular power spectrum of CMB anisotropies for spherical harmonics with index $\ell \lesssim 30$. A map of the calculated CMB sky (without smoothing) and the corresponding observation by the COBE DMR team are shown in Fig. 2. We have also calculated the fluctuations induced in the dark matter [22, 23, 24]. We found that the CMB and the dark matter fluctuation spectra have the correct, approximately scale invariant form. To reconcile the amplitude of the CMB anisotropies with that of the dark matter fluctuations, we have to introduce a bias factor of approximately

$$b = 2 \sim 3.$$

This means, we have to require that the observed fluctuations in the galaxy distribution are about a factor b larger than the corresponding dark matter fluctuations. This is a relatively high bias, but since the underlying non linear physical processes which produce the biasing (the difference in the clustering properties of light versus those of mass) are poorly understood, we don't want to draw strong conclusions from this result.

Our findings prompted us to search for a signature, which is due to purely linear clustering and which might lead to an observational distinction of defect models as compared to models of structure formation, where initial fluctuations are due to quantum fluctuations during inflation (inflationary models). As we outline in the next paragraph, acoustic peaks in the angular power spectrum of the CMB provide such a signature.

2.2 Acoustic Peaks

Due to the coherent acoustic oscillations of baryons and radiation prior to recombination (decoupling), there appear a series of peaks in the angular power spectrum of the CMB. The first peak is usually situated around $\ell \sim 200$, which corresponds to an angle of about 1^o, the horizon scale at the time of recombination. The subsequent peaks are damped by photon diffusion, but inflationary models predict a series of four to five peaks until $\ell \sim 1500$, where the damping washes them out completely. With a simple analytical model inspired by our simulations, we have estimated the position and height of the first peak for defect models. We found that the position is at $\ell = 350$ rather than at $\ell = 220$ and that the amplitude is lower than for standard inflationary models[25] (see Fig. 3).

This finding is very important also for observers, which will try to measure the height and the position of the peaks in the coming years [4, 26]. Therefore, we want to investigate it by thorough numerical simulations in the next years. For these simulations we can still determine the scalar field evolution as before. After recombination, we trace the the motion of photons in the gravitational field induced by the texture. In addition, we have to monitor the acoustic oscillations in the radiation/baryon fluid prior to decoupling. The decoupling era in principle has to be modeled by a Boltzmann code, but for a start, we mimic decoupling by an instantaneous transition from the strong coupling regime (fluid regime) to no coupling, free photons. The damping due to the finite mean free path during the decoupling era (Silk damping) is put in by hand.

Since it is impossible to investigate a dynamical range of more than about 20 in a 3d simulation (with a 256^3 grid), we will concentrate on the calculation of $50 \leq \ell \leq 1000$. It would however be very desirable to be able to calculate the the full CMB anisotropy spectrum, $2 \leq \ell \leq 1500$ in one single approach. The required dynamical range of about 700 would require at least a 7000^3 grid which is clearly out of range of any computers available within foreseeable future.

We therefore try to find also analytic approximations which model the power spectrum of the energy momentum tensor of the scalar field. These model spectra can then be used as sources in the linear perturbation equations for the gravitational field, the photons and the dark matter. If we further approximate the linear perturbation equations in k–space by equations for the power spectra, which depend only on the modulus $|\mathbf{k}|$, we can reduce the problem to about 2000 systems of coupled ordinary differential equations in 10 to 20 variables each. Such systems can easily be solved with present day resources.

We thus attack the problem of fully specifying the angular power spectrum of CMB anisotropies for defect models from two sides: On one side we perform solid 3d simulations with as little approximations and compromises as possible. On the other side, we try to find semi–analytical approximations which allow us to tackle a much wider dynamical range and to understand the physical processes taking place in more detail, but which always have to be checked for accuracy by full simulations.

3 Computation and Numerics

We have in the first place to simulate the (non–linear) time evolution of a multi component scalar field in 3 dimensions. The computational requirements and the numerical complexity of this task are considerable, but we believe it is absolutely necessary to be able to compare the predictions of the texture scenario with observations to sufficient accuracy.

We have calculated the time evolution of the texture field and its energy momentum tensor which we have split into scalar vector and tensor modes using fast Fourier transforms. Furthermore, we have calculated the induced density fluctuation spectrum of the dark matter. With the help of the linearized Einstein equations (our perturbation equations), we have determined the gravitational field of the texture and dark matter perturbations, which is used to calculate the anisotropies induced in the CMB (by means of the relativistic Liouville equation). In addition we want to simulate the baryon/radiation fluid prior to recombination.

Furthermore, we investigate approximations for the power spectra of the components of the energy momentum tensor of the scalar field.

So far, we use $N^3 = 200^3$ and $N^3 = 256^3$ grids for our simulations. We hope that the capacity of SX4 will allow us to extend to 512^3 grids.

As already mentioned, the main numerical problem is the time evolution of the scalar field. We solve the sigma model equations by directly minimizing the discretized action. We use a code which is second order accurate in time and space. One of the main tests of our scalar field code is energy momentum conservation. The components of the scalar field stress energy tensor are about 5% accurate on intermediate scales, scales much larger than the grid spacing, $\lambda > 10\Delta x$ but, considerably smaller that the full grid size, $\lambda < (N/4)\Delta x$.

The numerical methods to calculate the gravitational field, to trace the photons and to evolve the dark matter are straight forward and pose no numerical problem.

To determine the CMB anisotropies on smaller scales, $\theta < 2^o$, or, equivalently $\ell > 100$, we have to take into account the interactions of baryons and radiation before recombination. Before electrons and protons recombine to neutral hydrogen, baryons + radiation can be treated as a single tightly coupled fluid. During the recombination process the coupling gets gradually weaker and the problem can be treated by a Boltzmann equation. (At early times this is not possible since the frequent collisions render the Boltzmann equation too stiff to be solved numerically.)

The programs we use are mainly developed by ourselves in Fortran77. We also apply a real 3d FFT written by Andrea Bernasconi at CSCS to perform FFTs and the library routine ZUFALL to generate random numbers to set up the initial conditions.

To evaluate our results we use false color maps produced with the graphics package AVS at CSCS and we model our data with χ^2–minimization schemes and other statistical methods.

4 Conclusions

We show how problems in cosmology can be addressed with numerical simulations. The numerical complexity of the scalar field evolution with the non–linear first derivative terms in the σ–model equation of motion can at best be compared to the complexity of lattice gauge theories. The response of matter and radiation (CMB) to the geometry induced by the scalar field is determined in first order perturbation theory.

Satellite and ground based measurements of the angular power spectrum of CMB anisotropies up to angular scales of a few arc minutes ($\ell \sim 1000$) with an accuracy of about 1% will be performed in the coming years[26]. We want to be prepared and present the corresponding predictions from our theoretical models with comparable quality.

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Fig. 2

In 2a the observed COBE DMR map of the sky after subtraction of the dipole and of galactic contributions is shown. In 2b a simulated map of cosmic microwave background anisotropies due to texture defects (with somewhat higher angular resolution) is presented.

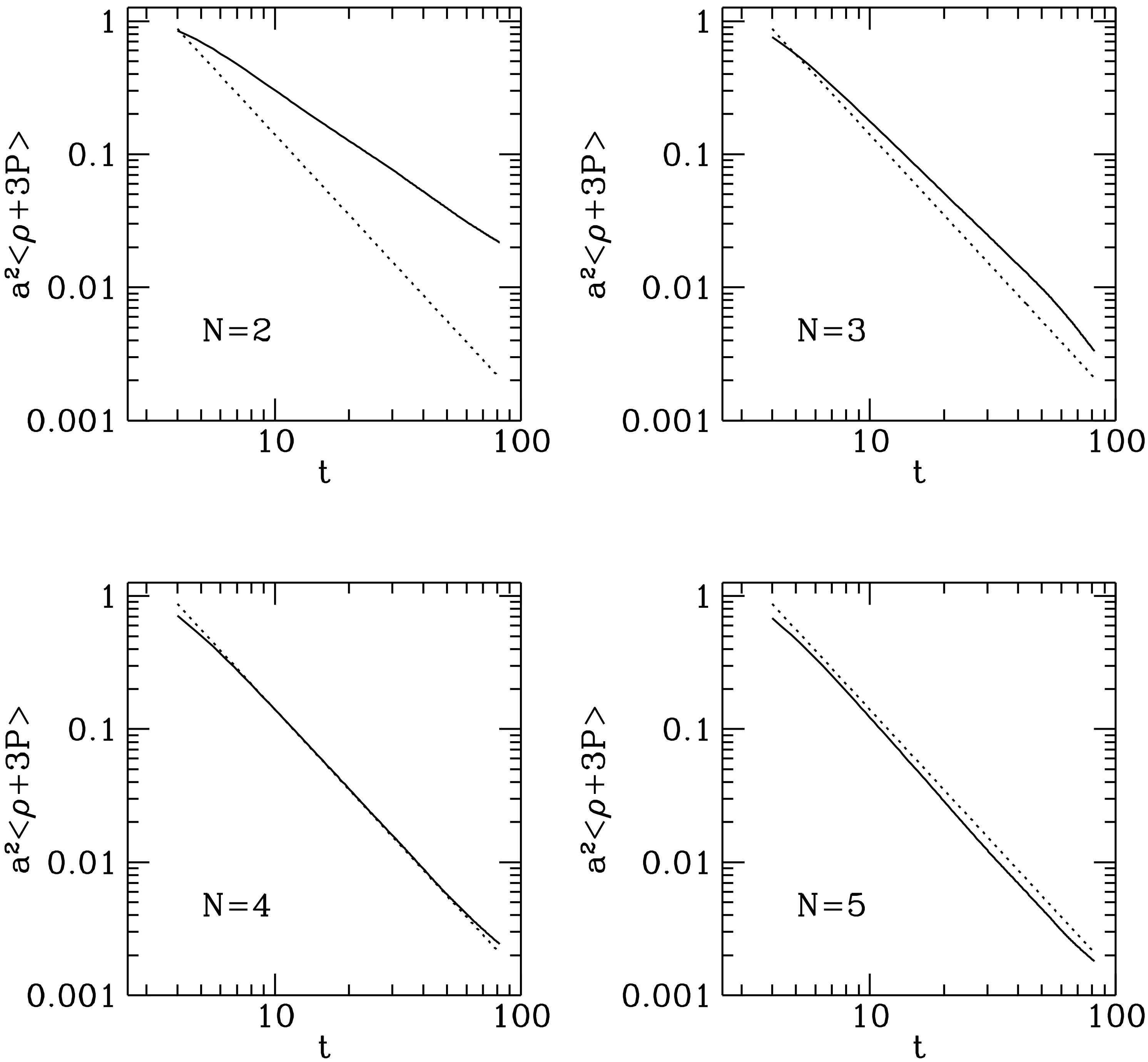

Figure 1: The scaling behavior for ρ found numerically in $(128)^3$ simulations for different $O(N)$ models is shown. The time is given in units of the grid spacing Δx. For comparison a dashed line $\propto 1/t^2$ is drawn. After some initial oscillations, for $N \geq 3$, ρ scales properly until $t \sim 80$, where finite size effects become important. For $N = 2$, global strings, scaling is probably violated by a logarithmic component.

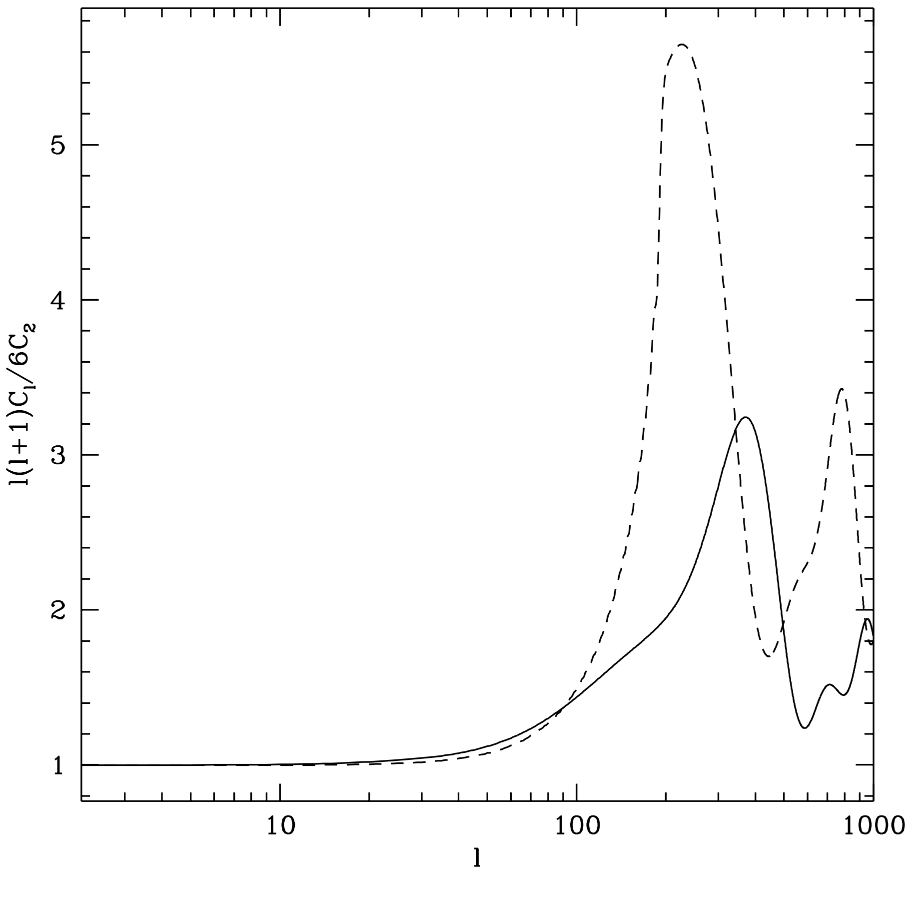

Figure 2: The solid line represents the angular power spectrum of cosmic microwave background anisotropies for a model with topological defects (texture). The dashed line shows the power spectrum for a standard inflationary model. Notice the difference in the position and height of the first peak. (Secondary peaks are calculated in a crude, not very accurate manner.)