THE COSMOLOGICAL CONSTANT AND GALAXY FORMATION

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Summary,. We derive a restrictive upper bound for the cosmological constant from the requirement that the formation of galaxies in a cold dark matter scenario should be compatible with the present observational limits for possible anisotropies of the microwave background. If the total density parameter Ω (including the vacuum energy) is equal to the critical value ($\Omega = 1$), we find for the contribution Ω_V of the vacuum energy density the conservative bound $\Omega_V < 0.7$.

1 Introduction

It is now generally recognized that the cosmological constant problem represents one of the deepest mysteries of fundamental physics and cosmology (see [Weinberg (1989)] for a review). The effective cosmological constant Λ obtains most likely contributions from short distance physics. For this reason is seems a miracle that micro–physics should be fine tuned with practically infinite precision just so that the universe can be big and flat. Such a fine tuning can hardly be due to symmetries since all the known symmetries — apart from the electromagnetic gauge group — are broken. An interesting large distance — small distance connection has recently been proposed in the framework of Euclidean quantum gravity [Banks (1988), Coleman (1988), Hawking (1984)], which leads to a distribution of the cosmological constant which is overwhelmingly concentrated at zero. It may, however, turn out that a more realistic treatment of wormholes in the Banks–Coleman–Hawking mechanism may produce a much less sharp peak around $\Lambda = 0$. Moreover, on a more fundamental level Euclidean quantum gravity (cosmology) poses many conceptual problems.

At any rate, the recent theoretical discussions led once more to the view that the cosmological constant Λ may very well be different from zero and that the corresponding vacuum energy density $\rho_V = \Lambda/(8\pi G)$ might at present even be much larger than the matter density ρ_m, and could, therefore, show up in astronomical observations. For instance, from number counts of galaxies as a function of redshift [Loh (1986)] derived quite a stringent bound for ρ_V/ρ_m [Loh and Spillar (1986)]. In this analysis it was, however, assumed that the luminosity distribution does not evolve with time. A re–analysis of the same data by Bahcall and Tremaine (1988) showed that a plausible model of galaxy evolution relaxes Loh's bound considerably.

In a paper which stimulated the present investigation, Weinberg (1987) derived an "anthropic" upper bound on Λ from the requirement that the formation of sufficiently large gravitational condensations should be possible. The idea of such an anthropic bound was originally formulated in the book of Barrow and Tipler (1986), where they derive a restrictive anthropic bound on a negative cosmological constant. The quantitative analysis for an $\Omega = 1$ universe led Weinberg to the conclusions that ρ_V might be more than two orders of magnitude larger than ρ_m. At the moment no astronomical data seem to be in contradiction to this possibility, if all the existing uncertainties are taken into account.

This result prompted us to ask whether such a large ratio ρ_V/ρ_m would be compatible with presently popular scenarios of galaxy formation. To be specific, we have studied this question for an $\Omega = 1$ universe, for which ρ_m is dominated by collisionless cold dark matter. It is well known that such a model for the special case $\Lambda = 0$, with primordial Gaussian fluctuations and biasing, is quite successful and has become a kind of "standard model" for galaxy formation. In particular the cold dark matter scenario is still compatible with the enormous isotropy of the microwave background radiation. It is to be expected that this constraint is violated, once Ω_V is sufficiently large. Indeed we show that the stringent anisotropy limits lead to a rather strong bound for ρ_V/ρ_m.

The sum $\rho_V + \rho_m$ is always assumed to be equal to the critical density, ρ_{crit}, for the well known reason that $\Omega = 1$ is an unstable fixed–point for the Friedman evolution. Hence any deviation from this fixed point — within the observational uncertainties — would require ridiculous fine tunings, even at the time of big bang nucleosynthesis.

2 Derivation of an upper bound for Ω_V

For a quantitative analysis we compute the ratio of the growth factors for density perturbations, from the time t_{eq} of equal matter and radiation (photons and neutrinos) densities to the present time t_0, for the following two scenarios:
(I) the "standard model" with $\Omega = 1$, $\rho_V = 0$ and a dominating cold dark matter (CDM) component. In scenario (II) we allow for a nonvanishing vacuum energy density ρ_V, while the matter density ρ_m is again assumed to be dominated by CDM. Our considerations could easily be extended to other models of galaxy formation.

Let $\Delta_+(t)$ be the growing mode for matter density fluctuations $\delta\rho_m/\rho_m$ as a function of cosmic time t. The growth factor A is defined by

$$A = \frac{\Delta_+(t_0)}{\Delta_+(t_{eq})} \ , \tag{1}$$

and we are interested in the ratio A_{II}/A_I for the two specified scenarios. This ratio is estimated within linear perturbation theory.

The growth factor A_I is well known:

$$A_I = 1 + z_{eq} \ . \tag{2}$$

For model II the differential equation for Δ remains the same, as can easily be shown. Setting $\tau = \sqrt{6\pi G\rho_V}t$, we have

$$\Delta'' + 2\frac{a'}{a}\Delta' = \frac{2}{3}(\rho_m/\rho_V)\Delta \tag{3}$$

and the Friedman equation reads

$$(a'/a)^2 = \frac{4}{9}(\frac{\rho_m}{\rho_V} + 1) \ , \tag{4}$$

which implies in the nonrelativistic regime

$$\rho_m/\rho_V \equiv x = (\sinh^2\tau)^{-1} \ . \tag{5}$$

The growing mode of (3) is then given by the monotonically increasing function

$$\Delta_+ = \text{const} \cdot x^{1/3} Q_{1/3}^{2/3}(\sqrt{1+x}) \ , \tag{6}$$

where $Q_{1/3}^{2/3}$ is the associated Legendre function of the second kind [Weinberg (1987)]. For the growth factor A_{II} the normalisation constant in (6) is irrelevant. Obviously $\tau_{eq} \ll 1$ and therefore $x_{eq} \gg 1$. This leads to the approximation

$$\Delta_+(\tau_{eq}) \approx x_{eq}^{-1/3} \ ,$$

if the normalisation in (6) is chosen such that we have asymptotically

$$\lim_{\tau\to\infty} \Delta_+ = \lim_{x\to 0} \Delta_+ = \frac{2}{\sqrt{\pi}}\Gamma(2/3)\Gamma(11/6) = 1.437 \ .$$

(See [Abramowitz and Stegun (1968)].) Thus

$$A_{II} \approx \Delta_+(x_0) x_{eq}^{1/3} \ . \tag{7}$$

(The subscript nought denotes always the value of a quantity at the present time t_0.) For $\Delta_+(x_0)$ we can in principle not use any simple analytic expression. But, as one can see in Figure 1, Δ_+ varies by less than a factor of 2 for $0 \leq x \leq 1.5$. Therefore, and for the sake of presentation, we will use the the inequality $\Delta_+(x_0) < \Delta_+(0) = 1.437$. We thus obtain

$$A_{II} < 1.437 x_{eq}^{1/3} .$$

This leads to an lower bound for x_0, whose improvement by an accurate numerical treatment is given afterwards. Next we use $x_{eq}^{1/3} = x_0^{1/3}(1 + z_{eq})$. (For $t < t_{eq}$ we can neglect ρ_V.) For three massless neutrino flavors we have, in standard notation, also

$$1 + z_{eq} \approx 2.5 \times 10^{-4} \Omega_m(t_0) h_0^2 . \tag{8}$$

Note that z_{eq} differs in the two scenarios under consideration.

Putting everything together, we obtain the inequality

$$A_{II}/A_I < 1.437 \times \frac{x_0^{4/3}}{1 + x_0} . \tag{9}$$

For the "standard model" (I) the fluctuations at recombination time are still compatible with the observational upper limits for the $\delta T/T$ fluctuations of the cosmic microwave background. The expected anisotropies are at least 20% of these observational limits at the relevant angular scales [Bond and Efstathiou (1984)],[Bond and Efstathiou (1987)], [Bernardis et al. (1988)], [Davies (1988)]. For this reason we must require $A_{II}/A_I \geq \epsilon$, $\epsilon \geq 0.2$. If this is combined with the result (9) we obtain the condition

$$\frac{x_0^{4/3}}{1 + x_0} \geq 0.7\epsilon \ , \ \epsilon \geq 0.2 . \tag{10}$$

This leads to the conservative bound $x_0 > 0.28$ or $\Omega_V < 0.8$ for $\epsilon = 0.2$. As can be seen from Figure 2, the accurate numerical treatment, where one replaces the upper limit 1.437 by the numerical value of $\Delta_+(x_0)$, does not significantly change this result: It leads to the somewhat stronger constraint

$$x_0 > 0.36 \ , \quad \text{or} \quad \Omega_V < 0.7 . \tag{11}$$

Thus, if we modify the standard cold dark matter scenario for galaxy formation by allowing a non–vanishing vacuum energy density with $\Omega_V + \Omega_m = 1$, the observational $\delta T/T$ constraints for the cosmic microwave background would be violated, unless $\Omega_V < 0.7$. This limit is substantially stronger than the anthropic bound of Weinberg (1987,1989) and can thus not be explained by an anthropic principle.

Finally we want to mention that similar considerations apply for low density universes ($\Omega < 1$) and for any contributions of cosmic "fluids" with negative pressure, $0 > p \geq -\rho$. The limitations of low density universes due to the compatability of the CDM scenario of galaxy formation with the isotropy of the cosmic microwave background is studied, for example, in [Bond and Efstathiou (1984)].

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Figure Captions

Figure 1: The growing mode of linear density perturbations in a CDM universe with cosmological constant is drawn as a function of $x = \rho_m/\rho_V$. The normalisation is chosen such that $\Delta_+(0) = 1.437$.

Figure 2: The quotient of the growth factors of linear density perturbations in a CDM universe with and without cosmological constant is drawn as a function of $x = \Omega^{(m)}/\Omega^{(vac)}$. The conservative limit $\delta T/T \leq 5(\delta T/T)_{CDM}$ is indicated.