

## Errata of the book “Gravitational Waves. Vol. 1”

Last update: Apr. 24, 2022.

Note: Corrections with a date earlier than Dec. 2013 have been implemented in the version that has been reprinted in Dec. 2013.

### A. Significant corrections

- Page 82, eq. (2.125), “ $\lambda_g > 300h_0$  kpc” should read “ $\lambda_g > 300h_0^{-1}$  kpc” and, in eq. (2.126), “ $m_g < 2 \times 10^{-29}h_0^{-1}$  eV” should read “ $m_g < 2 \times 10^{-29}h_0$  eV”. Four lines above eq. (2.125), “ $(1 - 10)h_0$  Mpc” should read “ $(1 - 10)h_0^{-1}$  Mpc”.

(11/11/13).

- Sect. 3.3.5, pages 121-124. The discussion in this section is not correct. The energy-momentum tensor given in eq. (3.121) is valid only for free particles, and is conserved only if all  $\dot{p}_A^i = 0$  for all particles. A priori, evaluating it on a pre-assigned trajectory such as the circular orbit of a binary system, is therefore not consistent. When we take into account the interaction, in a post-Newtonian expansion  $T^{\mu\nu}$  must be replaced by  $\tau^{\mu\nu}$ , which also contains potential terms, see e.g. (5.111)-(5.113), that ensure its conservation on the trajectories determined by the same potential. What allows us to use consistently the free-particle energy-momentum tensor to compute the mass quadrupole radiation for a binary system in a bound orbit is the fact that for a self-gravitating system the interaction terms, such as the gravitational potential  $-Gm_1m_2/r$ , are  $O(v^2)$ . Since the mass quadrupole  $M^{ij}$  is obtained from  $T^{00}$ , and  $T^{00}$  is  $O(v^0)$ , to lowest order the terms  $O(v^2)$  generated by the interaction can be neglected when we compute mass quadrupole radiation (and, to lowest order in the PN expansion, the conservation equation  $\partial_0 T^{00} + \partial_i T^{0i} = 0$  is satisfied independently of the trajectory). The same is true for mass octupole and current quadrupole radiation.

(27/12/11. Thanks to N. Johnson-McDaniel).

The correct text that replaces pages 121-124 in the revised printing is available as the file pages121-124.pdf, on the errata web page.

- Page 123, eq. (3.127). The second term in the last line is not there, and the correct result is:

$$M^{ij} = mx_{CM}^i x_{CM}^j + \mu x_0^i x_0^j$$

The discussion below this equation is therefore not correct. The correct statement is that, if the system is closed,  $x_{CM}$  is independent of time, and does not contribute to GW production.

(09/12/11. Thanks to T. Schoenenbach)

Again, the correct text is available in the file pages121-124.pdf

- Page 126, eq. (3.142). In the expression for  $M^{ijk}$  is missing a factor  $\delta m/m$ , where  $\delta m = m_2 - m_1$  and  $m = m_1 + m_2$ . It is derived observing that, in the CM frame, the positions of the two bodies are  $\mathbf{x}_1 = +(m_2/m)\mathbf{x}$  and  $\mathbf{x}_2 = -(m_1/m)\mathbf{x}$ . Therefore

$$T^{00}(t, \mathbf{x}) = m_1 c^2 \delta^{(3)}\left(\mathbf{x} - \frac{m_2}{m} \mathbf{x}_0(t)\right) + m_2 c^2 \delta^{(3)}\left(\mathbf{x} + \frac{m_1}{m} \mathbf{x}_0(t)\right),$$

and

$$\begin{aligned} M^{ijk}(t) &= \frac{1}{c^2} \int d^3x T^{00}(t, \mathbf{x}) x^i(t) x^j(t) x^k(t) \\ &= \mu \frac{\delta m}{m} x_0^i(t) x_0^j(t) x_0^k(t). \end{aligned}$$

(27/12/11).

- Page 127-128. A double dot is missing over all occurrences of  $J^{p,l}$  in eqs. (3.151)

(06/09/10. Thanks to E. Mitsou),

and a triple dot is missing over all occurrences of  $J^{p,l}$  in (3.152)

(12/06/13. Thanks to L. Philippoz).

- Page 139, eq. (3.211). In the third line, the term  $\epsilon_{ijk} S_{jL-1}^{(l+1)}(u) S_{kL-1}^{(l+1)}(u)$  should read  $\epsilon_{ijk} M_{jL-1}^{(l+1)}(u) S_{kL-1}^{(l+1)}(u)$ .

(05/05/21. Thanks to K. Fransen).

- Page 161-162. In the solution of Problem 3.3 there is a conceptual error: the quantity  $\dot{S}^{kl,m}$  that gives the sum of the mass octupole plus current quadrupole cannot be computed consistently using the energy-momentum tensor of the free particle and neglecting the contribution from the gravitational potential, as done in the text. The reason is that  $S^{kl,m}$  is given by the spatial components  $T^{kl}$  of the energy-momentum tensor, which are  $O(v^2)$  and therefore, for a self-gravitating system, they are of the same order as the interaction term due to the gravitational potential. In contrast, the mass octupole and the current quadrupole *can* be computed neglecting the potential terms, since they are obtained from  $T^{00}$  and  $T^{0i}$ , respectively. Since  $T^{00} = O(v^0)$  and  $T^{0i} = O(v)$ , the contribution of the gravitational potential to the mass octupole and to the current quadrupole is of higher order in  $v/c$ . The correct calculation should then be performed as follows: (1) starting from eq. (3.34), transform  $S^{kl}$  and  $\dot{S}^{kl,m}$  into the mass quadrupole, mass octupole and current quadrupole using (3.52) and (3.54). Since the derivation of these equations makes use of the exact energy-momentum conservation, here the gravitational potential terms in  $T^{\mu\nu}$  are automatically taken into account. Then (2): compute the mass quadrupole, mass octupole and current quadrupole using the free-particle energy-momentum tensor, since here the inclusion of the gravitational potential term would give a higher-order contribution. It is instead incorrect to use the free-particle energy-momentum tensor directly into  $\dot{S}^{kl,m}$ . Unfortunately, in Problem 3.3, after correctly computing the mass octupole radiation, I computed the current quadrupole radiation by evaluating first the contribution from  $\dot{S}^{kl,m}$  with the free-particle energy-momentum tensor (which is wrong) and subtracting from it the mass octupole. Once the computation is performed correctly one finds that the current quadrupole radiates only at the frequency  $\omega_s$  (rather than at  $\omega_s$  and at  $3\omega_s$ , as the mass octupole). Furthermore, one recovers exactly the result obtained in Eqs. (5.266-5.267) from the full relativistic Blanchet-Damour approach (the reason being that also the relativistic corrections to the orbit are of higher order).

(27/12/2011. Thanks to N. Johnson-McDaniel)

A revised version of this Problem can be found in the file pages161-162.pdf, in the errata web page.

- Page 163, eq. (3.354), “ $\mathbf{3} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{1}$ ” should be “ $\mathbf{3} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{0}$ ”.  
(4/12/13).
- Page 172, second line, “ $R_0^2 = \dots$ ” should be “ $R_0^3 = \dots$ ”.  
(25/12/2021. Thanks to Emmanuele Battista)
- Page 194, 6 lines above the end of the section, “Up to distances of order 600 Mpc” should read “Up to distances of order 25 Mpc”.  
(13/6/08).
- Page 201, eq. (4.211), in the second line “ $1 + \dots$ ” should read “ $(I_1 + I_2)/2 + \dots$ ” and, in eq. (4.213), “ $1 - \dots$ ” should read “ $(I_1 + I_2)/2 - \dots$ ”.  
(22/11/10. Thanks to J. Romano and M. Normandin)
- Page 245, eq. (5.47). The correct expression for the energy-momentum tensor of a set of free particles in curved space is

$$T^{\mu\nu} = \frac{1}{\sqrt{-g}} \sum_a m_a \frac{d\tau_a}{dt} \frac{dx_a^\mu}{d\tau_a} \frac{dx_a^\nu}{d\tau_a} \delta^{(3)}(\mathbf{x} - \mathbf{x}_a(t)),$$

where  $c^2 d\tau_a^2 = -g_{\mu\nu} dx_a^\mu dx_a^\nu$ .

(4/12/13)

- Page 246, eq. 5.50, “ $2\phi c^2$ ” should read “ $\phi c^2$ ”.  
(9/7/08)
- Page 246, three lines below eq. 5.52: “The total action of the system is the sum over all particles, ...”. Actually, one must also add the contribution from the Einstein-Hilbert action. A detailed explicit derivation (provided by Justin Vines) can be found in the file EIH Lagrangian.pdf, on the errata web page.  
(22/3/17. Thanks to J. Vines)
- Page 318, eq. (6.67). In the last term in square bracket “ $-Gm/r$ ”, should read “ $+Gm/r$ ”.  
(10/6/08. Thanks to S. Foffa)

- Page 344, eq. (7.43). In the first two lines,  $\hat{s}(t)$  should be  $\hat{s}$  ( $\hat{s}$  is a quantity already integrated in time, see eq. (7.41), and no longer depends on  $t$ ).  
(18/3/16)

## B. Minor typos

- Page 7, in eq. (1.23),  $d^4x$  should read  $d^4y$ .  
(08/07/13. Thanks to Y. Lay)
- Page 7, after eq. (1.26), “ $(1/c^2)\partial_0^2$ ” should read “ $\partial_0^2$ ”.  
(29/03/10. Thanks to J. Enander)
- Page 12, first line “Problem 1.1” should read “Problem 2.1”.  
(27/12/2011. Thanks to N. Johnson-McDaniel).
- Page 18, eq. (1.81),  $d\xi^i/d\tau$  should be  $d\xi^j/d\tau$ .  
(22/10/2023).
- Page 33, eq. (1.123). On the right-hand side,  $\bar{T}^{\mu\nu}$  should be  $\bar{T}_{\mu\nu}$  (lower indices, as in the other terms of the equation). One line above the equation, again  $\bar{T}^{\mu\nu}$  should be  $\bar{T}_{\mu\nu}$ , and one line below,  $\bar{T} = \bar{g}_{\mu\nu}\bar{T}^{\mu\nu}$  should be  $\bar{T} = \bar{g}^{\mu\nu}\bar{T}_{\mu\nu}$  and “by definition,  $\bar{T}^{\mu\nu}$ ...” should read “by definition,  $\bar{T}_{\mu\nu}$ ...”.  
Since in this section we are writing everywhere the Einstein equations with lower indices, we need to define a smoothed energy-momentum tensor  $\bar{T}_{\mu\nu}$  with lower indices. One can similarly define  $\bar{T}^{\mu\nu}$  by writing eq. (1.123) with all upper indices. Note that, since  $\bar{g}^{\mu\nu}$  only has low frequencies, it can be carried inside the average in eq. (1.123), and therefore  $\bar{T}^{\mu\nu} = \bar{g}^{\mu\rho}\bar{g}^{\nu\sigma}\bar{T}_{\rho\sigma}$ .  
(18/6/2019. Thanks to Yuntao Bai for a question that stimulated this correction)
- Page 36, after eq. (1.135), “ $\partial_t = (1/c)\partial_0$ ” should read “ $\partial_t = c\partial_0$ ”  
(29/03/10. Thanks to J. Enander)

- Page 37, second line : “to trow it” should read to “throw it”  
(12/06/13. Thanks to L. Philippoz)
- Page 68, end of note 11 : “where  $\mathbf{x}_2 - \mathbf{x}_2 = \mathbf{x}$ ” should read “where  $\mathbf{x}_2 - \mathbf{x}_1 = \mathbf{x}$ ”  
(12/06/13. Thanks to L. Philippoz)
- Page 82. The reduced Compton wavelength  $1/m_g$  is sometimes denoted  $\lambda_g$  and sometimes  $\lambda'_g$ .  
(4/12/13).
- Page 85, two lines below eq. (2.134), “two solution” should read “two solutions”
- Page 92, eq. (2.173),  $\delta\theta^2$  should be  $d\theta^2$ .  
(4/12/13).
- Page 96, line after eq. (2.189) : “the matrix whole elements” should read “ the matrix whose elements”  
(12/06/13. Thanks to L. Philippoz)
- Page 106, eq. (3.30): the derivatives are with respect to  $t$ , not with respect to  $x_0 = ct$ .  
(09/12/11. Thanks to T. Schoenenbach)
- Page 121, footnote 24: “Straumann (2003)” should be “Straumann (2004)”  
(12/06/13. Thanks to T. Schoenenbach).
- Page 122, eq. (3.123),  $\int d^3x x^i x^i(\dots)$  should be  $\int d^3x x^i x^j(\dots)$   
(12/06/13. Thanks to L. Philippoz)
- Page 149, two lines after eq. (3.273), “ $\mathbf{T}_{lm}^{E2}$  and  $\mathbf{T}_{lm}^{E2}$ ” should read “ $\mathbf{T}_{lm}^{E2}$  and  $\mathbf{T}_{lm}^{B2}$ ”  
(23/11/14).

- Page 186, eq. (4.120), the first equality “ $L = ma^2\omega_0 = \dots$ ” should read “ $L = \mu a^2\omega_0 = \dots$ ”.  
(13/3/12. Thanks to S. Dinkgreve).
- Page 208, six lines below eq. (4.252), the letter “i” should actually be a iota,  $0 \leq \iota \leq \pi$ .  
(12/06/13. Thanks to L. Philippoz)
- Page 221, caption of 4.19, “the the star” should be “the star”  
(4/12/13).
- Page 241, eq. (5.11),  $dt^2$  should be  $c^2 dt^2$ . The same in Note 10.  
(19/08/13).
- Page 245, eq. (5.48):  $\sum_i {}^{(2)}g_{ij}$  should read  $\sum_i {}^{(2)}g_{ii}$ .  
(11/01/2012. Thanks to N. Johnson-McDaniel)
- Page 246, eqs. (5.56): in all previous equations, eg. (5.49)–(5.52), I used  $\sum_{a \neq b}$  as a compact notation for the sum over  $a$  and over  $b$ , with the condition  $a \neq b$ . In the third term in eq. (5.56), I wrote instead explicitly  $\sum_a \sum_{b \neq a}$ . For consistency, it should have been written more simply as  $\sum_{a \neq b}$ , as in the other similar occurrences.  
(11/04/2022. Thanks to Emmanuele Battista)
- Page 252, 13th line before the end : “the GWs compute at” should read “the GWs computed at”  
(12/06/13. Thanks to L. Philippoz)
- Page 267, line after eq.(5.145) :  $\Lambda_{ijab}$  should be  $\Lambda_{ij,ab}$   
(12/06/13. Thanks to L. Philippoz)
- Page 275. In the subtitle to 5.3.5, “Radiation radiation” should read “Radiation reaction”.  
(08/05/12. Thanks to T. Schoenenbach).
- Page 275. In the sentence below eq. (5.180), “up to second in  $v/c$ ” should read “up to second order in  $v/c$ ”.  
(08/05/12. Thanks to T. Schoenenbach).

- Page 281. In eq. (5.199) “m!” should be “l!”  
(08/05/12. Thanks to T. Schoenenbach).
- Page 286, 4th line after eq. (5.224) : “expecially ” should read “especially”  
(12/06/13. Thanks to L. Philippoz)
- Page 301, fourth bullet point: “Scäfer” should be “Schäfer”  
(12/06/13. Thanks to T. Schoenenbach, and my apologies to Gerhard Schäfer).
- Pag. 304-330, in the header of the left pages “[...] incompact binaries” should be “[...] in compact binaries” (a weird LaTeX bug).  
(4/12/13)
- Page 305, 9th line before the end: “which can very dramatically” should read “which can vary dramatically”  
(12/06/13. Thanks to L. Philippoz)
- Page 317. In eqs. (6.62) and (6.64)  $\Delta_R$  should be replaced by  $c\Delta_R$ .  
(13/3/12. Thanks to S. Dinkgreve).
- Page 320. In eq. (6.90)  $\Delta_R$  should be replaced by  $c\Delta_R$  (while eq. (6.95) is correct since  $\Delta_S$  and  $r \equiv Gm/c^3$  both have dimensions of time)  
(13/3/12 and 12/6/13. Thanks to S. Dinkgreve and L. Philippoz).
- Page 331, 5th point of the Further reading section: “This corrections” should be “This correction”  
(12/06/13. Thanks to T. Schoenenbach).
- Page 351, note 17, “multipole detectors” should be “multiple detectors”  
(27/04/20)
- Page 388, line 2, “esplicitly” should be “explicitly”  
(26/05/20. Thanks to Pau Amaro Seoane).
- Page 391, eq. (7.183),  $f_0^{-4/3}$  should read  $f_0^{-2/3}$ .  
(19/12/16. Thanks to Yota Watanabe).



- Page 471, two lines below eq. (9.3), “ $2L_x = \dots$ ” should read “ $2L_y = \dots$ ”  
(26/8/08. Thanks to P. Zimmerman)  
and, in eq. (9.4), the overall minus sign should not be there.  
(25/2/16. Thanks to S. Sello).
- Page 477 line before eq. (9.36): “The equation of the geodesic equation” should be “The equation of the geodesic deviation”  
(12/06/13. Thanks to T. Schoenenbach).
- Page 494, three lines below eq. (9.129), “ $h_{xx} = h_+$  and  $h_{xx} = -h_+$ ” should read “ $h_{xx} = h_+$  and  $h_{yy} = -h_+$ ”.  
(25/1/10. Thanks to B. Aylott)
- Page 515, below eq. (9.205): “which is smaller that the size” should be “which is smaller than the size”  
(12/06/13. Thanks to T. Schoenenbach).
- Page 520, line 9,  $E_\gamma = |\mathbf{p}|/c$  should read  $E_\gamma = |\mathbf{p}|c$ . The subsequent factors of  $c$  are correct.  
(29/02/20. Thanks to Lorenzo Aiello).
- Bibliography: “S. L. Finn” should read “L. S. Finn”.  
(27/12/2011. Thanks to N. Johnson-McDaniel, and my apologies to Lee Samuel Finn)

I will be glad to receive further corrections from readers.