Self-Referred Decision Rules and Chaos

A. Corcos[1], *J.-P. Eckmann*[2,3], *and A. Malaspinas*[2]

[1]CEFIB, Université Paris 2, France
[2]Dépt. de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
[3]Section de Mathématiques, Université de Genève, CH-1211 Genève 4, Switzerland

The stock market crash of October 1987 has drawn widespread attention which has focused on the role played by mimetic behavior—that is, self-referred decision rules—on market prices. The aim of this paper is to present a model which takes into account the effects of collective opinions. The presence of mimetic behavior for the determination of prices has of course been studied earlier—Arthur (1987), Bikhchandani *et al.*(1992), Orléan (1986, 1990, 1992), Shiller (1984), Topol (1991), West (1988). But the numerous economic considerations to which our model applies are quite specific, and new: it generates in a purely mechanical way self-fulfilling *speculative bubbles* whose collapse is intrinsic to the model. In particular, the dynamics of prices is completely deterministic, and the chaotic behavior has *purely* deterministic origin.

Before explaining and analyzing the model in subsequent sections, we want to compare it in three respects to current theories of economic behavior.

1. Since the model operates on a purely deterministic basis, it actually challenges the external and unpredictable origin of market prices. Our model exploits the continuous mimicry of financial markets to show that the disordered and random aspect of the time series of prices can be explained not only by the advent of "random" news and events, but can just as well be generated by the behavior of the agents fixing the prices.

The dynamics of prices in our model is deterministic and derives from the theory of chaotic dynamical systems, which have the feature of exhibiting endogeneously perturbed motion. After the first papers on the theory of chaotic systems, such as Lorenz (1963), May (1976), (see, e.g., Collet-Eckmann (1980) for an early exposition), a series of economic papers dealt with models mostly of growth—Benhabib-Day (1981), Day (1982, 1983), Stutzer (1980). Later, a vast and varied number of fields of economics was analyzed in the light of the theory of chaos—Grandmont (1985, 1987), Grandmont-Malgrange (1986). They extend from macro-economics—business cycles, models of class struggles, political economy—to micro-economics—models with overlapping generations, optimizing behavior—and touch subjects such as game theory and the theory of finance. The applicability of these theories has been thoroughly tested on the stock market prices—Brock *et al.*(1987), Brock (1988), Brock-Dechert (1988), LeBaron (1988), Brock *et al.*(1991), Hsieh (1989), Scheinkman-LeBaron (1989a, 1989b)—in studies which tried to detect signs of non-linear effects and to nail down the deterministic nature of these prices. While the theoretical models—Van Der Ploeg (1986), De Grauwe-Vansanten (1990), De Grauwe *et al.*(1993)—seem to agree on the relevance of chaotic deterministic dynamics, the empirical studies—Eckmann *et al.*(1988), Hsieh-LeBaron (1988), Hsieh (1989, 1991, 1992), LeBaron (1988), Scheinkman-LeBaron (1989a,b)—are less clear-cut, mostly because of lack of sufficiently long time series (Eckmann-Ruelle (1992)), or, because the deterministic component of market behavior is necessarily overshadowed by the inevitable external effects.

The model presented here shows a mechanism of price fixing—decisions to buy or sell dictated by comparison with other agents—which is at the origin of an instability of prices. From one period to the next, and in the absence of information other than the anticipations of other agents, prices can continuously exhibit erratic behavior and never stabilize, without diverging. Thus, the model questions the fundamental hypothesis that equilibrium prices have to converge to the intrinsic value of an asset.

2. We can also consider our model in the context of the increasing market volatility of financial markets. The volatility of prices generated by our chaotic model could give a beginning of an explanation of the excess volatility observed on financial markets—Grossman-Shiller (1981), Fama (1965), Flavin (1983), Shiller (1981), West (1988)—which traditional models, such as ARCH, try to incorporate (Engle (1982), Bollerslev *et al.*(1991), Bollerslev (1987)).

3. Finally, we can see speculative bubbles in our model as a natural consequence of mimetism. We can compare this to the two basic trends in explaining the problem of bubbles. The first makes reference to rational anticipations—Muth (1961)—and rests on the hypothesis of efficient markets. With fixed information, and knowing the dynamics of prices, the recurrence relation for the price is seen to depend on the fundamental value and a self-referential component, which tends to cause a deviation from the fundamental value: this is a speculative bubble—Blanchard-Watson (1982). This theory of rational speculative bubbles fails to explain the birth of such events, and even less their collapse, which it does not predict either. The second trend purports to explain speculative bubbles by a limitation of rationality—Shiller (1984), West (1988), Topol (1991). It allows to incorporate notions which the neo-classical analysis does not take into account: asymmetry of information, inefficiency of prices, heterogeneity of anticipations—Grossman (1977), Grossman-Stiglitz (1980), Grossman (1981), Radner (1972, 1979). In our approach, which follows the second trend, the agents act without knowing the actual effect of their behavior: this contrasts the position of a model-builder—Orléan (1986, 1989, 1990, 1992). This, in turn can lead to prices which disconnect from the fundamental indicators of economics.

In the present paper an attempt is being made to show that self-referred behavior in financial markets can generate chaos and speculative bubbles. They will be seen to be caused by mimetic behavior: bubbles will form and collapse when certain agents believe in the advent of a turn of trend, while they observe the behavior of their peers.

1. The Model

We consider an economy in which the population makes portfolio choices between a non-risky asset and a risky one when tomorrow's price is uncertain. The choice depends on expected capital gains. The portfolio choice then becomes a price expectation problem. Each agent has a different set of information, obtained by observing other agents. Agents do not operate with reference to fundamental value, but rather with respect to *expected market price*. They are able to make profits if their expectations are judiciously chosen. It is rational for the agent—Keynes (1936), Orléan (1986, 1989)—to take into account collective judgements in order to make portfolio profits.

That is why, for constituting expectations at time $t+1$, the information used by an agent is the price expectation, at time t, of a certain sample of *other* agents randomly chosen among the population. This takes into account collective opinion and its expected correctness, that is, their confidence (or absence thereof) in the continuation of a deviation from the fundamental value. Their opinion refers to two kinds of price, market price and fundamental value, as exhibited by Keynes (1936):

1. Speculation relying on short term action and especially market opinion and market price. The most important aspect is the market price expectation, that is, the collective opinion about future market prices.
2. Firm behavior: long term behavior relying on economic reality and fundamental value. This leads agents to detect excessive increase or decrease of market price and thus leads to anticipatory adaptation of the market price. This causes the collapse of the bubble.

These well-known principles generate different kinds of risks between which agents choose by arbitrage. The former is a *competing risk*—Keynes (1936), Orléan (1989)—which leads agents to imitate the collective point of view since the market price includes it. Thus, it is assumed that Keynes' animal spirits may exist. More simply, there is the risk of mistaken expectation: agents believe in a price different from the market price.

On the other hand, in the latter case, the emerging price is not necessarily in harmony with economic reality and fundamental value. Self-referred decisions and self-validation phenomena can then indeed lead to speculative bubbles or sunspots—Azariadis (1981), Azariadis-Guesnerie (1982), Blanchard-Watson (1982), Jevons (1871), Kreps (1977). Thus, the latter risk is the result of *precaution*. It addresses the fitting of market price to fundamental value, and by extension, collapse of the speculative bubble.

Both attitudes are likely to be important and are integrated in decision rules. Agents realize an arbitrage between the two kinds of risk we have described. That is why they have both a mimetic behavior and an antagonistic one: they either follow the collective point of view or they have reversed expectations.

We are now going to put these assumptions into mathematical form. We assume that, at any given time t, the population is divided into two parts. Agents are explicitly differentiated as being bullish or bearish in proportion p_t, and $q_t = 1 - p_t$, respectively. The first ones expect an increase of the price, while the bearish ones expect a decrease. The agents then form their opinion for time $t+1$ by sampling the expectations of other agents at time t, and modifying their own expectations accordingly. More precisely, in our model, a bullish agent will sample

m other agents. This is a first parameter in our model. We then introduce threshold densities ρ_{hb}, ρ_{hh}. We assume $0 \leq \rho_{hb} \leq 1, 0 \leq \rho_{hh} \leq 1$. A bullish agent will change opinion if at least one of the following propositions is true:

1. At least $m \cdot \rho_{hb}$ among the m agents inspected are bearish.
2. At least $m \cdot \rho_{hh}$ among the m agents inspected are bullish.

The first case corresponds to "following the crowd," while the second case corresponds to the "antagonistic behavior." The quantity ρ_{hb} is thus the threshold for a bullish agent ("haussier") to become bearish ("baissier") for mimetic reasons, and similarly, ρ_{hh} is the threshold for a bullish agent to become bearish because there are "too many" bullish agents. The thresholds $m \cdot \rho_{hb}$ and $m \cdot \rho_{hh}$ which are proportional to the sample size m, are a measure of the risk aversion of the bullish population. Large ρ_{hb}, ρ_{hh} mean that the agent will rarely change opinion. He is risk-adverse and would like to see an almost unanimity appearing before changing his mind. In the opposite case, if the thresholds are close to 0.5, agents like risk and believe already in a weak trend. Putting the above rules into mathematical equations we see that the probability P for an agent who is bullish at time t to change his opinion at time $t+1$ is:

$$P = \text{Prob}\left(\{x < m \cdot (1 - \rho_{hb})\} \cup \{x > m \cdot \rho_{hh}\}\right) ,$$

where x is the number of bullish agents found in the sample of m agents. In an entirely similar way, we introduce thresholds ρ_{bh}, and ρ_{bb}, and the probability Q for a bearish agent at time t to become bullish at time $t+1$ is:

$$Q = \text{Prob}\left(\{x < m \cdot (1 - \rho_{bh})\} \cup \{x > m \cdot \rho_{bb}\}\right) .$$

We can combine these two rules into a *dynamical law* governing the time evolution of the populations. Denoting p_t the proportion of bullish agents in the population at time t, we can find the new proportion, p_{t+1}, at time $t+1$, by taking into account those agents which have changed opinion according to the *deterministic law* given above. To simplify notation, we let $p_{t+1} = p'$ and $p_t = p$. Then, the above statements are easily used to express p' in terms of p, by using the probability of finding j bullish people among m (Corcos (1993)):

$$\begin{aligned} p' &= p - p \cdot \sum_{\substack{j \geq m \cdot \rho_{hb} \\ \text{or } j \leq m \cdot (1 - \rho_{hh})}} \binom{m}{j} p^{m-j} (1-p)^j \\ &\quad + (1-p) \cdot \sum_{\substack{j \geq m \cdot \rho_{bh} \\ \text{or } j \leq m \cdot (1 - \rho_{bb})}} \binom{m}{j} (1-p)^{m-j} p^j \qquad (1.1) \\ &\equiv F_{\underline{\rho},m}(p) , \end{aligned}$$

where $\underline{\rho} = \{\rho_{hb}, \ \rho_{bh}, \ \rho_{hh}, \ \rho_{bb}\}$. Thus, the function $F_{\underline{\rho},m}(p)$ completely characterizes the dynamics of the proportion of bullish and bearish populations.

2. Analysis

The law given by Equ.(1.1) is not easy to analyze, and we give in Fig. 1 a few sample curves $F_{\underline{\rho},m}$. We see that as m gets larger, the curves seem to tend to a limiting curve. Using this observation, our conceptual understanding of the dynamics can be drastically simplified if we consider the problem for a large number of polled partners, m. Indeed, it is most convenient to first study the unrealistic problem $m = \infty$ and to view the large m case as a perturbation of this limiting case. The main ingredient in the study of the case $m = \infty$ is the Law of Large Numbers, which we use in a form given in Feller (1966):

Lemma. *Let g be a continuous function on* $[0, 1]$. *Then, for $s \in [0, 1]$,*

$$\lim_{m\to\infty} \sum_{j=0}^{m} \binom{m}{j} s^j (1-s)^{m-j} \cdot g(j/m) \;=\; g(s)\;.$$

We apply this lemma to the (piecewise continuous) function $g = f_h$, where f_h is the indicator function of the set defining P:

$$f_h \;=\; \begin{cases} 1\,, & \text{if } x \geq \rho_{hb} \text{ or } x \leq 1-\rho_{hh}, \\ 0\,, & \text{otherwise}\;. \end{cases}$$

Similarly, we define

$$f_b \;=\; \begin{cases} 1\,, & \text{if } x \geq \rho_{bh} \text{ or } x \leq 1-\rho_{bb}, \\ 0\,, & \text{otherwise}\;. \end{cases}$$

It is now easy to check that the lemma implies

$$\lim_{m\to\infty} F_{\underline{\rho},m}(p) \;=\; p - p\cdot f_h(1-p) + (1-p)\cdot f_b(p) \;\equiv\; G_{\underline{\rho}}(p)\;.$$

Note again that we do not consider $G_{\underline{\rho}}(p)$ itself as an evolution law for the population of bullish agents, but $G_{\underline{\rho}}$ can serve very well as an approximation for the true laws $F_{\underline{\rho},m}$ for large m. In Fig. 1 we show how the functions $F_{\underline{\rho},m}$ converge to $G_{\underline{\rho}}$.

3. Discussion of the Dynamics

In the preceding section, we have shown how to gain a qualitative understanding of the maps $F_{\underline{\rho},m}$, when m is large. We can now apply in a rather straightforward way the general theory of 1-dimensional discrete time dynamical systems (see e.g., Collet-Eckmann (1980)) to the functions $F_{\underline{\rho},m}$. The recurrence $p_t \to p_{t+1}$ can exhibit several typical behaviors which, for large m depend essentially only on $\underline{\rho}$. We enumerate a few of them and refer the reader to Figs. 2 and 3.

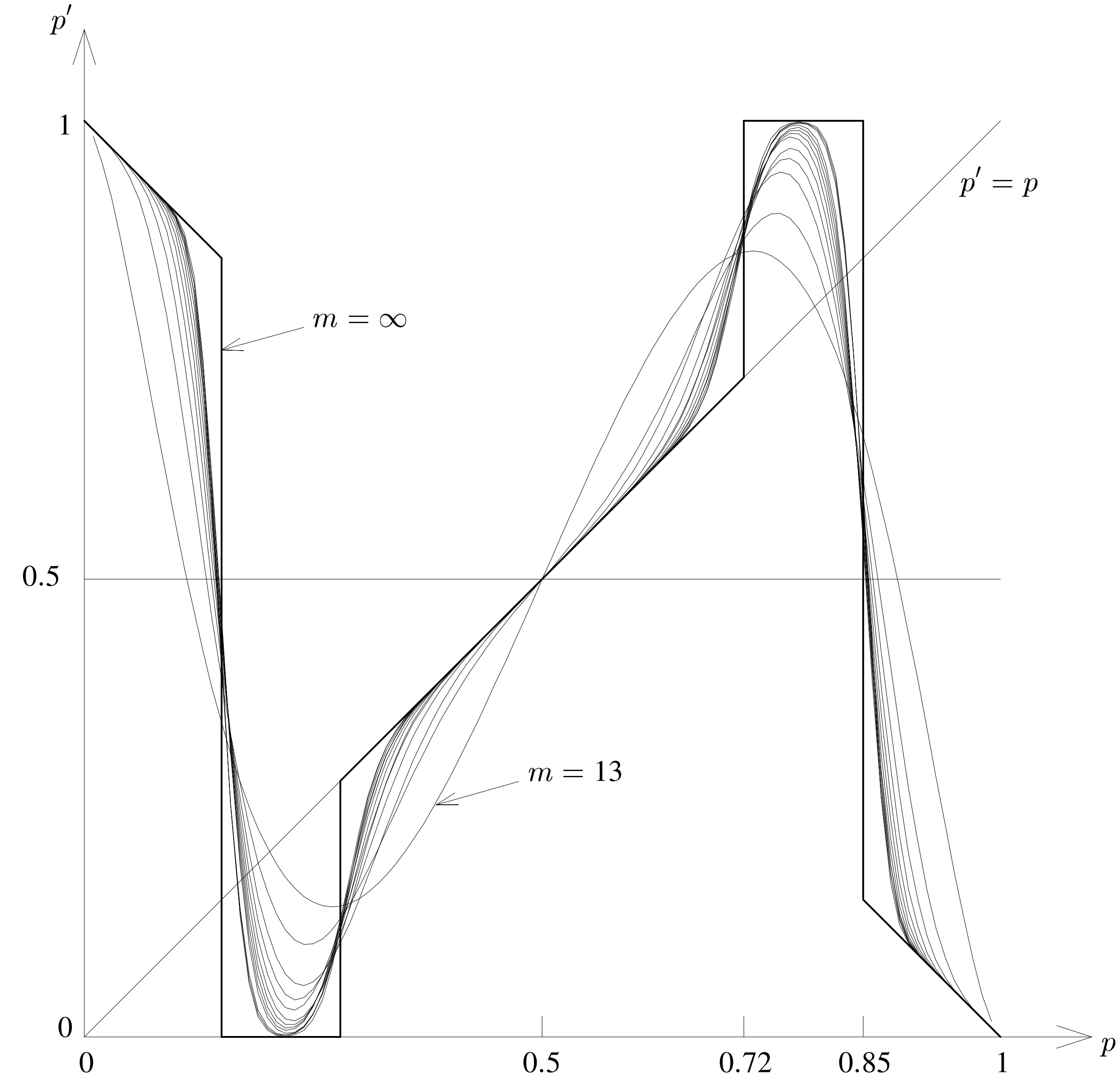

Fig. 1: The family of functions $F_{\underline{\rho},m}$, for $\rho_{hh} = \rho_{bb} = 0.85$, and $\rho_{hb} = \rho_{bh} = 0.72$. The curves are for $m = 13 + j \cdot 26$, $j = 0, \ldots, 13$. Note the convergence to the function $G_{\underline{\rho}}$, (indicated by $m = \infty$).

1. The most trivial case is the appearance of a stable fixed point. This will occur for example for $\rho_{hh} = \rho_{bb} = 0.75$, $\rho_{hb} = \rho_{bh} = 0.72$, and $m = 60$. Then, the population will equilibrate, and converge to $p = 1 - q \approx 0.68$, or to $p \approx 0.32$.
2. The next more interesting case is the appearance of a limit cycle (of period 2): at successive times, the population of bullish and bearish agents will oscillate between 2 different values. This happens, e.g., for $\rho_{hh} = \rho_{bb} = 0.76$, with the other parameters as before.
3. But for certain values of the parameters, e.g., $\rho_{hh} = \rho_{bb} = 0.85$, the sequence of values of p_t is a *chaotic* sequence, with positive Liapunov exponent (cf. Eckmann-Ruelle (1985)). The mechanism for this is really a sufficiently strong combination of mimetic and antag-

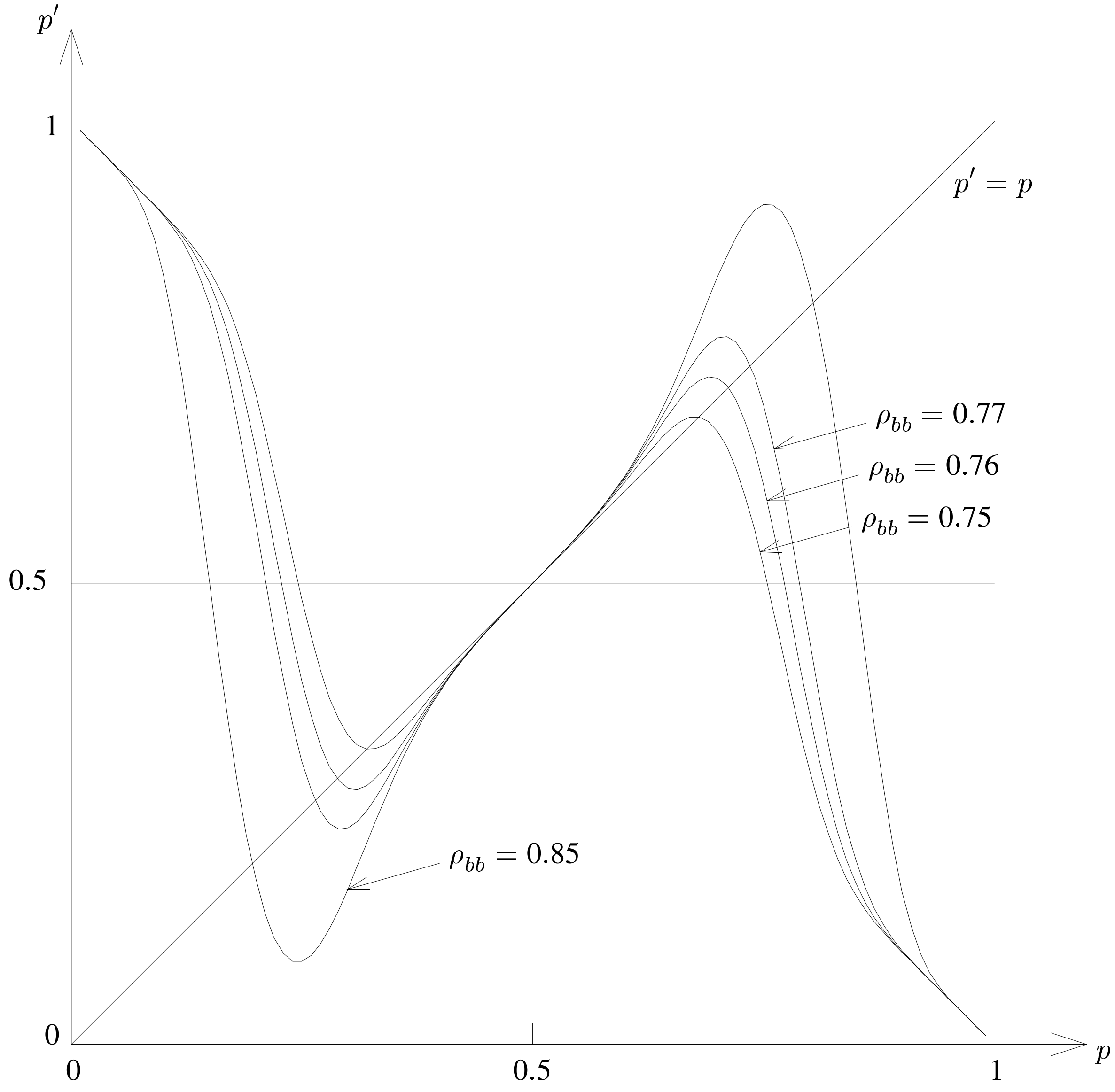

Fig. 2: Four curves $F_{\underline{\rho},m}$, for $m = 60$ and $\rho_{hb} = \rho_{bh} = 0.72$, with $\rho_{hh} = \rho_{bb} = 0.75, 0.76, 0.77, 0.85$.

onistic behavior, which is typical of any population of agents. It is this *self-referential* behavior of the anticipations *alone* which is responsible for a deterministic, but seemingly erratic evolution of the population of bearish agents. No external noise is needed to make this happen, and in general, we view external stimuli as acting on top of the intrinsic mechanism which we exhibit here (Eckmann (1981)). Note that the set of parameter values $\underline{\rho}$ for which chaos is expected (say, near the values used at the bottom of Fig. 3) has positive Lebesgue measure.

We next consider in more detail the time evolution of p_t for the parameter values of the last frame of Fig. 3, which are typical for the abundant set of "chaotic" parameter values, and we will

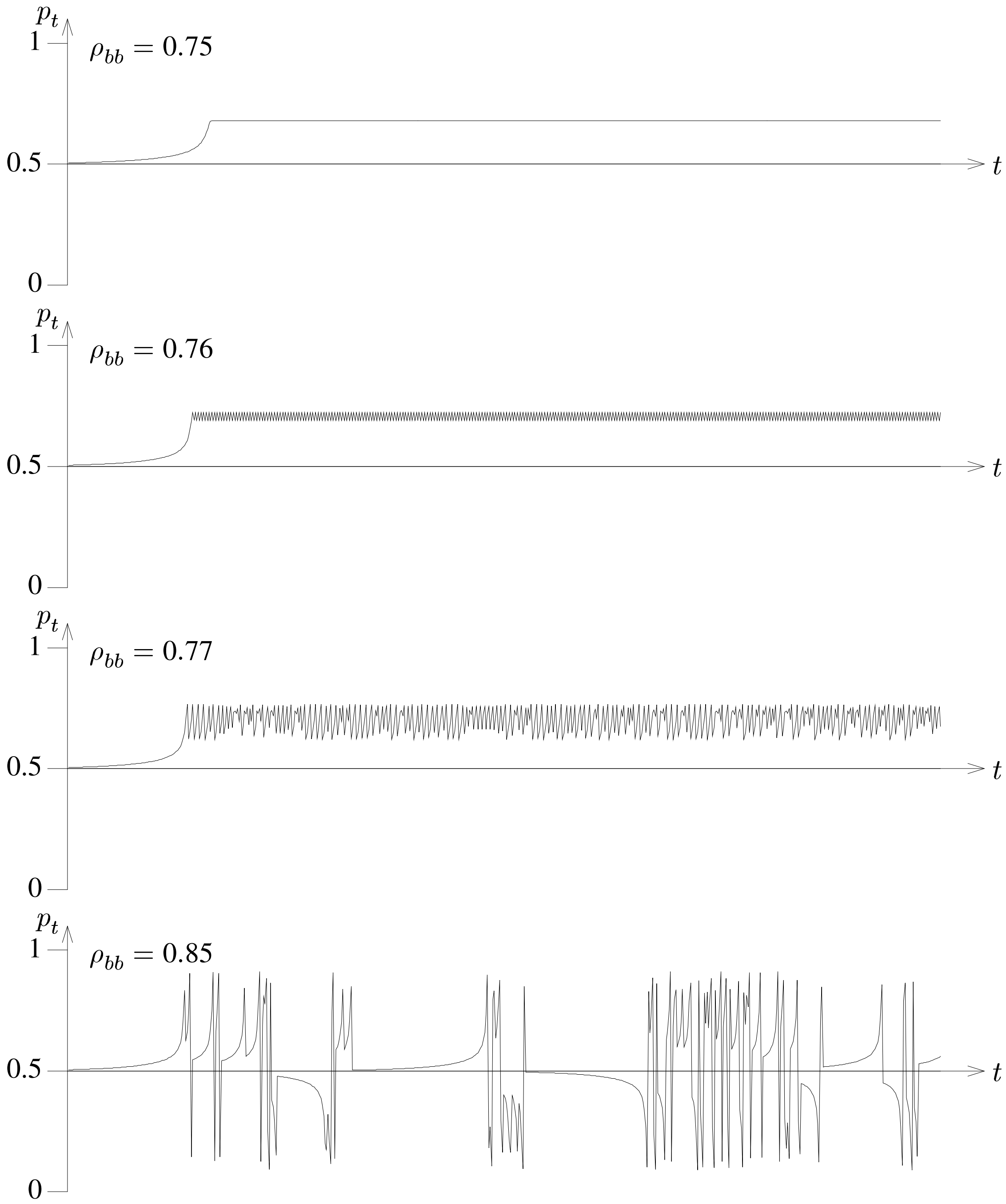

Fig. 3: The time series for the same parameter values as in Fig. 2. Note that for $\rho_{hh} = \rho_{bb} = 0.75$ one has convergence to a bullish equilibrium, for 0.76 a bullish period 2, for 0.77 a bullish, but chaotic behavior. The most interesting case is $\rho_{hh} = \rho_{bb} = 0.85$, where calm periods alternate in a seemingly random fashion with speculative bubbles.

show how the time evolution exhibits "speculative bubbles." This phenomenon is akin to the notion of intermittency (of "Type I") as known to physicists, see e.g., Manneville (1991) for an exposition. Indeed, we can distinguish two distinct behaviors in the last frame of Fig. 3, which occur repeatedly with more or less pronounced separation. The first process is the "laminar phase," which is seen to occur when the population p_t is near 0.5. Then, the evolution of the population is slow, and the population grows slowly away from 0.5, either monotonically or through an oscillation of period 2, depending on $\underline{\rho}$. This motion is slower, when the inspected sample size (m) is larger, reflecting a more stable evolution for less independent agents. When the distance from 0.5 is large, erratic behavior sets in, which persists until the population reaches again a value of about 0.5, at which point the whole scenario repeats. The determinism of the model is reflected by "equal causes lead to equal effects," while its chaotic nature is reflected by the erratic length of the laminar periods, as well as of the bubbles of wild behavior.

Having analyzed the evolution of the number of bullish agents, we next describe how the price π_{t+1} of an asset at time $t + 1$ is related to the proportion p_t of bullish agents. One can argue (Corcos(1993)) that the price change $\pi_{t+1} - \pi_t$ from one period to the next is a *monotone* function of p_t (and, perhaps, of π_t). This function is positive when $p_t > 1/2$ and negative when $p_t < 1/2$. If the reaction to a change in p_t is reflected in the prices in the next period, then a bubble in p_t will lead to a speculative bubble in the prices in the next period. Thus, our model predicts the occurrence of bubbles from the behavior of the agents alone. Furthermore, for quite general laws of the form

$$\pi_{t+1} = H(\pi_t, p_t) ,$$

a simple application of the chain rule of differentiation leads to the observation that the variable π_t has the same Liapunov exponent as p_t. In fact, this will be the case if $0 < \partial_\pi H < \lambda$ and $\partial_p H > c > 0$, where λ is the Liapunov exponent for p_t, as follows from $\delta\pi_{t+1} = \partial_\pi H \cdot \delta\pi_t + \partial_p H \cdot \delta p_t$. This condition is, in particular, satisfied for a law of the form $\pi_{t+1} = \pi_t + G(p_t)$, where G is strictly monotone. Thus, chaotic behavior of bullish agents leads to chaotic behavior of prices.

To summarize: the interesting feature of all this is that we observe a time evolution which, while satisfying certain criteria of randomness (such as possessing an absolutely continuous invariant measure and exhibiting a positive Liapunov exponent—cf. Eckmann-Ruelle (1985)) at the same time exhibits some regularities on short time scales, since it is deterministic. Our model thus establishes that straightforward fundamental conditions may suffice to generate chaotic stock market behavior, depending on the parameter values. If the market adjusts present market price on the basis of expectations and mimicry—self-referred behavior—then chaotic evolution of the population will also imply chaotic evolution of prices.

4. Conclusions

The traditional concept of stock market dynamics envisions a stream of stochastic "news" that may move prices in random directions. This paper, in contrast, demonstrates that certain types of deterministic behavior—mimicry and contradictory behavior alone—can already lead to chaotic prices.

If the market prices are assumed to follow the p_t behavior, our description refers to the well-known evolution of the speculative bubbles. Such apparent regularities often occur in the stock market and form the basis of the so-called "technical analysis" whereby traders attempt to predict future price movements by extrapolating certain patterns from recent historical prices. Our model provides an explanation of birth, life and death of the speculative bubbles in this context.

While the traditional theory of rational anticipations exhibits and emphasizes self-reinforcing mechanisms, without either predicting their inception nor their collapse, the strength of our model is to justify the occurrence of speculative bubbles. It allows for their collapse by taking into account the combination of mimetic and antagonistic behavior in the formation of expectations about prices.

The specific feature of the model is to combine these two Keynesian aspects of speculation and enterprise and to derive from them behavioral rules based on collective opinion: the agents can adopt an imitative and gregarious behavior, or, on the contrary, anticipate a reversal of tendency, thereby detaching themselves from the current trend. It is this duality, the continuous coexistence of these two elements, which is at the origin of the properties of our model: chaotic behavior and the generation of bubbles.

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