## Testing Lorentz Invariance Violation with WMAP Five Year Data

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We consider different renormalizable models of Lorentz invariance violation. Based on birefringent photon propagation of cosmic microwave background photons generically induced in such models, we constrain Lorentz invariance violation using the five year data of the Wilkinson microwave anisotropy probe (WMAP). We derive limits on a birefringent effective photon mass and on a polarization dependence of the speed of light which are significantly stronger than other published limits.

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The principle spacetime symmetry of particle interactions in the standard model is Lorentz invariance. Experiments confirm Lorentz symmetry at currently accessible energy scales of up to 2 TeV. This scale will be extended shortly to 14 TeV with Large Hadron Collider at CERN. Although present experiments confirm Lorenz invariance to a good precision, it can be broken in the very early Universe when energies approach the Planck scale. There are a number of extensions of the standard model of particle physics and cosmology that violate Lorentz invariance (for reviews see Refs. [1–5]).

As it can be expected, Lorentz invariance violation (LV) affects photon propagation (the dispersion relation), and generically results in a rotation of linear polarization (birefringence). Other effects include new particle interactions such as a photon decay and vacuum Cherenkov radiation [4]. All these effects can be used to probe Lorentz invariance. The dispersion measure (DM) test is based on a phenomenological energy dependence of the photon velocity ([6], see also Refs. [7] for reviews and Refs. [8–10] for recent studies of this effect; early discussions include Refs. [11]; Refs. [6, 9, 10] consider LV models which preserve rotational and translational invariance but break boost invariance).

Several models of LV predict frequency dependent effects. For discussions of such high energy LV see Refs. [12–14]. Refs. [15] study generalisations of electromagnetism, motivated by this kind of Lorentz invariance violation. On the other hand, LV associated with a Chern-Simons interaction [16, 17] affects the complete spectrum of electromagnetic (EM) radiation, not just the high frequency part, and induces a frequency independent rotation of polarization (see Sec. 4 of Ref. [2]).

To determine the effects induced by LV, it is useful to apply an analogy with the propagation of electromagnetic waves in a magnetized plasma as outlined in Refs. [8, 12, 16, 18, 19]. In this formalism it is easy to see that for LV models which depend not only on frequency but also on polarization, the rotation measure (RM) constrains the symmetry breaking scale more tightly than pure DM (see Refs. [14, 15, 19]).

The propagation of ultra-high energy photons represents a promising possibility to probe Lorentz symmetry [20]. Gamma Ray Bursts (GRB) are astrophysical objects located at cosmological distances which emit very energetic photons [6] (for reviews of cosmological tests involving GRBs, see Refs. [3, 21]). Testing LV through RM by analysis of GRB polarization is proposed in [22, 23], after the observation of highly linearly-polarized  $\gamma$ -rays from GRB021206 has been reported [24]. Although this measurement has been strongly contested [25], there is evidence that the  $\gamma$ -ray flux from GRB930131 and GRB960924 is consistent with more than 35% and 50%polarization, respectively [26]. However, the issue of polarization of GRB  $\gamma$ -rays is still under debate and additional X-ray studies are needed to either confirm or disprove polarization of GRB  $\gamma$ -rays [27].

In this letter we mainly consider renormalizable models of LV described in Ref. [2]. We use the very well understood and measured temperature anisotropy and polarization of the cosmic microwave background (CMB) to constrain LV. These data have been proposed as a probe of Lorentz invariance in the Universe in Refs. [28, 29]. In our study we use the WMAP 5 year results [31] and obtain limits which are significantly more stringent than those obtained from radio galaxy polarimetry [16]. As we shall see, generic LV is birefringent, i.e. depends on the photon polarization. This leads to a rotation of the CMB polarization which induces parity-odd cross correlations, such as Temperature-B-polarization and E-Bpolarization. These correlators vanish in models which preserve parity. Generally speaking, the effect is similar to that induced by a homogeneous magnetic field [30].

Let us consider an electromagnetic wave with frequency  $\omega$  and spatial wave vector  $\mathbf{k}$ ,  $k \equiv |\mathbf{k}|$ . A linearly polarized wave can be expressed as superposition of left (L) and right (R) circularly polarized waves. In a magnetized plasma, a homogeneous magnetic field induces a difference in the phase velocity of L and R waves. Therefore it causes a rotation of the polarization (called Faraday rotation [33]). The group velocity of the wave also differs from c. These two effects can be expressed in terms of the refractive indices defined by  $k_{\rm L,R} = n_{\rm L,R}\omega$  where  $k_{\rm L,R}$  denotes the wave number for L(-) and R(+) waves. The indices  $n_{\rm L,R}$  are [33]

$$n_{\rm L,R} = \sqrt{\varepsilon_1 \mp \varepsilon_2} \tag{1}$$

Here the upper (lower) sign corresponds to the L (R) polarization and  $\varepsilon_1$  and  $\varepsilon_2$  are components of the electric permittivity or dielectric tensor  $\varepsilon_{ij}$ ,  $\varepsilon_1 = \varepsilon_{xx} = \varepsilon_{yy} = 1 + \omega_p^2/(\omega_c^2 - \omega^2)$  and  $\varepsilon_2 = \varepsilon_{yx} = -\varepsilon_{xy} = (\omega_c/\omega)\omega_p^2/(\omega_c^2 - \omega^2)$ , where  $\omega_p$  and  $\omega_c$  are the plasma and electron cyclotron angular frequencies (see Sec. 4.9 of Ref. [33]).

The magnitude of both DM, due to the different group velocity<sup>1</sup> and RM, the rotation of polarization, are proportional to the photon travel distance  $\Delta l$ ,

$$\Delta t_{\rm L,R} = \Delta l \left( 1 - \frac{\partial k_{\rm L,R}}{\partial \omega} \right), \qquad (2)$$

$$\Delta \alpha = \frac{1}{2} (k_{\rm L} - k_{\rm R}) \Delta l.$$
 (3)

Here,  $\Delta t_{\rm L,R}$  is the difference between the L (R) travel time and that of a "photon" traveling at the speed of light, and  $\Delta \alpha$  is the polarization rotation angle.

Faraday rotation is widely used in astrophysics to limit magnetic fields in galaxies and clusters (see Ref. [34] for a review and references therein). In cosmology, Faraday rotation of CMB photons [30, 35] has been used to constrain the amplitude of a homogeneous as well as a stochastic cosmological magnetic field [36, 37]. In the following, we show that LV leads to a modification of the Maxwell equations [14, 15] analogous to the modifications described above.

Following Ref. [2], the most general renormalizable form of LV can be expressed by two additional terms in the action (we set  $\hbar = 1$ )

$$\Gamma_{\rm LV} = \int d^4x \left[ K_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho} - \frac{1}{4} L^{\mu} \epsilon_{\mu\nu\lambda\rho} A^{\nu} F^{\lambda\rho} \right], \quad (4)$$

where Greek indices  $(\mu, \nu, \lambda, \rho)$  denote time-space coordinates,  $\epsilon_{\mu\nu\lambda\rho}$  is the totally antisymmetric tensor,  $F^{\mu\nu}$ is the electromagnetic field tensor,  $A^{\nu}$  is the vector potential,  $L^{\mu} = (L^0, \mathbf{L})$  has the dimension of mass and describes a super-renormalizable (dimension 3) coupling,  $K_{\mu\nu\lambda\rho}$  is a renormalizable, dimensionless coupling giving raise to a dimension 4 operator.  $K_{\mu\nu\lambda\rho}$  has the same symmetries as the Riemann tensor and we only consider its trace-free part which is analog to the Weyl tensor<sup>2</sup>. Both terms in Eq. (4) lead to birefringence but the frequency dependence is different.

The first term in the action  $\Gamma_{\rm LV}$  can be computed within Newman-Penrose formalism [2]. The dispersion relation is given by [2]

$$\omega^2 = k^2 \pm 8\omega^2 |\Psi_0| \,. \tag{5}$$

Here  $\Psi_0$  is the analog of the Newman-Penrose scalar (for more details see [2]),  $\Psi_0 = -\left[K_{0i0j} - K_{0ilj}n^l - K_{kilj}n^kn^l\right]m^im^j$ , where **m** and  $\bar{\mathbf{m}}$  represent the left and right circular polarization basis vectors and  $\mathbf{n} = \mathbf{k}/k$  is the photon propagation direction. Latin indices indicate spatial components of a vector or tensor.

The second term in the action  $\Gamma_{\rm LV}$  leads to the dispersion relation [2, 16]

$$(k_{\mu}k^{\mu})^{2} + (k_{\mu}k^{\mu})(L_{\nu}L^{\nu}) - (L_{\mu}k^{\mu})^{2} = 0, \qquad (6)$$

where  $k^{\mu}$  is time-space wave vector,  $(k^{\mu}) = (\omega, \mathbf{k})$ . To first order in the small parameters  $L_0$  and  $|\mathbf{L}|$  one has

$$\omega^2 = k^2 \pm k(L_0 - L\cos\phi),\tag{7}$$

where  $L = |\mathbf{L}|$ ,  $\phi$  is an angle between the photon propagation direction and the vector  $\mathbf{L}$ ,  $\cos \phi = (\mathbf{L} \cdot \mathbf{n})/L$ . Comparing Eq. (7) with the dispersion relation  $\omega^2 = k^2/(\varepsilon_1 \mp \varepsilon_2)$ , we find  $L_0 - L \cos \phi \simeq \omega_p^2/(\omega_c - \omega)$ .

To be general as possible, we rewrite the dispersion relation for the both types of LV in the form (see [19]),

$$\omega^2 = k^2 \left[ 1 \pm \left( \frac{M}{M_{\rm PL}} \right) \left( \frac{k}{M_{\rm PL}} \right)^{N-4} \right], \qquad (8)$$

where  $M_{\rm PL}$  is the Planck mass,  $M_{\rm PL} \simeq 1.2 \times 10^{19}$  GeV, N is the dimension of the Lorentz symmetry violating operator and M is a mass scale of the model. For N = 4 we then have  $8\Psi_0 = M/M_{\rm PL}$ , and for N = 3  $M = L_0 - L \cos \phi$ . Generally speaking our aim is to limit the function  $\gamma(k) \equiv \left(\frac{M}{M_{\rm PL}}\right) \left(\frac{k}{M_{\rm PL}}\right)^{N-4}$  from CMB birefringence. This ansatz can also be applied to nonrenormalizable models with higher dimension operators, N > 4.

To compute the CMB polarization rotation angle induced by LV, we follow the analogy with photon propagation in a magnetized medium which yields  $n_{L,R} = 1 \pm \gamma(k)/2$ . Using Eq. (3), we obtain

$$\Delta \alpha^{(LV)} = \frac{1}{2} \omega \gamma(k) \Delta l. \tag{9}$$

<sup>&</sup>lt;sup>1</sup> This modification may be viewed as an effective photon "mass" that makes the photon speed smaller (larger) than the speed of light c = 1 for R (L) waves and  $\omega < \omega_c$ .

 $<sup>^2</sup>$  The trace part also leads to DM but not to birefringence, we therefore do not consider it here.

In the case N = 4,  $\Delta \alpha^{(LV)}$  grows linearly with frequency. In such a model, and for all models with higher dimension operators, the best limits can in principle be obtained from high frequency photons (for example GRB  $\gamma$ -rays [22, 23]), while CMB photons are less affected. However, the fact that the theory of CMB anisotropies and polarization yields that both TB and EB polarization have to vanish in standard cosmology, while the polarization of GRB's is still under debate, at present, a test using CMB data is to be preferred. Another advantage is that for the CMB the distance  $\Delta l \simeq H_0^{-1}$  is maximal. In the dimension 3 model,  $\Delta \alpha^{(LV)} = -\frac{1}{2}(L_0 -$ 

In the dimension 3 model,  $\Delta \alpha^{(LV)} = -\frac{1}{2}(L_0 - L\cos\phi)\Delta l$ , is frequency-independent. In Ref. [16] the above result is applied to polarization data from distant radio galaxies,  $\Delta \alpha < 6^{\circ}$  at 95% C.L. at redshift  $z \sim 0.4$ . The constraint obtained if Ref. [16] is  $|L_0 - L\cos\phi| \leq 1.7 \times 10^{-42} h_0$  GeV, where  $h_0 \simeq 0.7$  is the present Hubble parameter in units of 100 km s<sup>-1</sup> Mpc<sup>-1</sup>.

Using recent WMAP results [31] and assuming Gaussian errors, we obtain the following limits on the absolute value of rotation angle

$$|\Delta \alpha|_{\rm obs} \le 4.90^{\circ}$$
 at 95% C.L. (10)  
 $|\Delta \alpha|_{\rm obs} \le 2.52^{\circ}$  at 68% C.L. (11)

We adopt  $\Delta l \simeq 9.8 \times 10^9 h_0^{-1}$  years. We express our results in terms of  $\nu_{100} = \nu/10^{11}$ Hz to keep them as independent of the CMB observation band frequency as possible. We also introduce an effective photon mass by the modified dispersion relation  $\omega^2 = k^2 \pm m_{\gamma}^2$ , i.e.

$$m_{\gamma}^2 = \omega^2 \gamma(k) = M \omega \left(\frac{\omega}{M_{\rm PL}}\right)^{N-3} = 2 \frac{\Delta \alpha}{\Delta l} \omega$$

For 4D- and 3D-models we have  $m_{\gamma}^{(4D)} = 2\omega |2\Psi_0|^{1/2}$  and  $m_{\gamma}^{(3D)} = [\omega(|L_0 - L\cos\phi|)]^{1/2}$  respectively. We compare our estimates with those of Ref. [22]; The cases considered here corresponds to the LV spectral indices as: for 3D model m = -1 and m = 0 for 4D model. Neither of such an index has been studied in Ref. [22].

Using Eq. (9), we find the following limit on the function  $\gamma$ :

$$\gamma(\omega_{\rm cmb}) \le 8.6 \times 10^{-31} \nu_{100}^{-1} h_0 \text{ at } 95\% \text{ C.L.},$$
 (12)

and an almost twice better limit at 68% C.L. From this we derive the constraint on the effective birefringent mass,

$$m_{\gamma} \le 3.8 \times 10^{-19} \left( h_0 \nu_{100} \right)^{1/2} \text{eV} \text{ at } 95\% \text{ C.L.}$$
 (13)

Note that left and right handed photons have effective masses of opposite sign. This result is model independent because  $m_{\gamma}$  only depends on the directly measured rotation angle  $\Delta \alpha$  and on the frequency.

We can also express the limit on  $\gamma$  in terms of a limit for the mass scale M or the dimensionless parameter  $M/M_{\rm Pl}$ 

$$\frac{M}{M_{\rm Pl}} \lesssim 10^{-31+33(N-4)} \nu_{100}^{3-N} h_0 \ . \tag{14}$$

For N > 4, the limits are not very interesting, while for N = 4 or N = 3 'naturally expected' values of the parameters are ruled out. More precisely, for the models considered we constrain the dimensionless scalar  $\Psi_0$  for the 4D model,  $|\Psi_0| \leq 1.1 \times 10^{-31} h_0 \nu_{100}^{-1}$  at 95% C.L., while we find for the 3D model  $|L_0 - L \cos \phi| \leq 3.6 \times 10^{-43} h_0 \text{GeV}$  at 95% C.L. This is almost one magnitude better than the limit obtained in Ref. [16].

If  $L \ll L_0$ , we can safely neglect the angular dependence, and assume that  $m_{\gamma}^{(3D)} = \sqrt{\omega L_0}$ . However, if  $L \gg L_0$ , the modification of the photon dispersion becomes direction dependent, and must be averaged over all sky for the CMB photons. Then, the rotation angle can be estimated by the two-point correlation function, *i.e.*,  $\Delta \alpha_{\text{eff}} = \langle |\Delta \alpha|^2 \rangle^{1/2}$ . A very rough estimate leads to a pre-factor  $\sim 1\sqrt{2}$ . In a more detailed analysis the presence of  $\mathbf{L}$  breaks rotational symmetry and leads to off-diagonal correlations in the temperature anisotropy and polarization spectra analog to the CMB polarization Faraday rotation by a constant magnetic field [35]. To take this fully into account requires to estimate the CMB Temperature-B polarization, E- and B-polarization cross correlations, as well as B-polarization spectra due to the LV vector field **L**, and to compare theoretical estimates with the CMB corresponding anisotropy data. Also the scalar  $|\Psi_0|$  of the 4D model breaks rotational symmetry and taking the direction dependence of  $\Delta \alpha$  into account is relatively complicated. We shall address this issue in future work, but it is expected that the resulting limits will not change significantly.

The obtained bound on a birefringent effective photon mass is nearly three orders of magnitude better than the limit given by the particle data group [38],  $m_{\gamma} \leq 2 \times$  $10^{-16}$  eV, but less stringent than those obtained from the limits on magnetic field generation [39] which are, however model dependent. Another useful bound is the departure of the refraction index in vacuum from unity, *i.e.*,  $|\Delta n| = |1 - k/\omega| = |\gamma(k)|/2$ . In the 4D model,  $|\Delta n^{(4D)}| \simeq 4|\Psi_0|$ , In the 3D model,  $|\Delta n^{(3D)}| \simeq L_0/2\omega$ (when  $L \ll L_0$ ). Generically Eq. (12) implies  $|\Delta n| \leq 4.3 \times 10^{-31} h_0 \nu_{100}^{-1}$ . The difference of the refractive index from 1 can be viewed as a difference of the photon speed from 1,  $\Delta c$  at the level of  $10^{-30} - 10^{-31}$ , which is much more stringent than the (more general) limit obtained in Ref. [40], which is  $\Delta c < 10^{-23}$ . The formalism given here is applicable for higher dimension operators too, but due to the frequency dependence  $|\alpha^{(LV)}| \propto \omega^{N-3}$  the CMB data based limits become much weaker that those given from high energy photons propagation ( $\gamma$  or Xrays). Even the bounds obtained from the Crab Nebulae radiation polarimetry looks to be more promising [41]

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