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Gravitational Radiation of Cosmic String Loops

Ruth Durrer[1]

Princeton University
Physics Department
Jadwin Hall
POB 708
Princeton NJ 08544
USA

Abstract

We discuss gravitational radiation of cosmic string loops in flat background. After presenting a general formula for the gravitational angular momentum radiation of localized periodic sources, we calculate the radiation of energy, momentum and angular momentum for some classes of loop configurations (one with cusps and another without cusps but with kinks). We find that the angular momentum radiated always points opposite to the angular momentum of the string itself.

Finally we investigate some cosmological consequences of our results.

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RUTH DURRER

Princeton University, Department of Physics: Joseph Henry Laboratories, Jadwin Hall, POB 708, Princeton NJ 08544, USA

1 INTRODUCTION

In this conference we heard a lot about the evolution of cosmic string networks in a Friedman universe (see contributions by Albrecht & Turok (AT), Benett & Bouchet (BB) and Shellard et al. in this proceedings). But up to now we did not discuss another important ingredient for the fate of cosmic strings: gravitational radiation.

Cosmic string loops are topologically unstable, i.e., they can decay. For the non superconducting, local cosmic strings discussed in this talk gravitational radiation is the most effective energy loss mechanism, as long as the curvature radius of the string is much larger than its thickness.

For several examples energy and momentum radiation ($\dot{E}$ and $\dot{\mathbf{P}}$) have already been calculated by Vachaspati & Vilenkin (1985) and Garfinkle & Vachaspati (1987), but the radiation of angular momentum, $\dot{\mathbf{L}}$, has never been investigated. In this work we thus especially emphasize angular momentum radiation.

There have been arguments (Vachaspati & Vilenkin 1985) that the 'rocket–effect' proposed by Hogan & Rees (1984) and Hogan (1987), i.e. acceleration due to radiation of transversal momentum, can be substantially reduced by angular momentum radiation which might rotate the direction of $\dot{\mathbf{P}}$. We shall find in Section 3 that $\dot{\mathbf{L}}$ is always anti–parallel to the angular momentum of the string. $\mathbf{L}^{(\mathbf{st})}$ is thus only diminished by angular momentum radiation but not rotated. Therefore, also the angle between $\dot{\mathbf{P}}$ and $\mathbf{L}^{(\mathbf{st})}$ remains constant, i.e. $\dot{\mathbf{P}}$ is not rotated. This, and the result that the velocity of the loop never becomes so small that dynamical friction has to be taken into account (Durrer 1989) lead us to the conclusion that typical cosmic string loops do show a substantial rocket–effect . This result is important for models of cosmological structure formation with cosmic string loops (see Section 4).

In Section 2 we discuss the general formalism for the treatment of gravitational radiation of periodic sources, and we present our results on gravitational angular momentum radiation (Durrer 1989). In Section 3 we apply this formalism on cosmic string loops and discuss numerical results for two classes of loops; one with cusps and one without cusps but with kinks. (The notion of cusps and kinks is explained in the introductory talk by T.W.B. Kibble.) Finally, we investigate some cosmological consequences of our results in Section 4.

2 GRAVITATIONAL RADIATION OF PERIODIC SOURCES

2.1 Gravitational Radiation of Isolated Systems

For isolated systems with asymptotically flat geometry, one can define an energy momentum 'tensor', $t_{\alpha\beta}$, $t_\alpha \equiv t_{\alpha\beta}\theta^\beta$. Although $t_{\alpha\beta}$ does not transform like a tensor, the integrals of $*t_\alpha$ over spacelike hypersurfaces make physical sense.
$(\theta^\beta)_{\beta=0}^3$ are an orthonormal, asymptotically Lorentzian basis of one forms and $*$ denotes the Hodge dual.

To obtain a conserved angular momentum in the usual way, we must require $t_{\alpha\beta}$ to be symmetric. We therefore adopt the Landau–Lifshitz energy momentum tensor which can be given in the form

$$*t^\alpha = -\frac{1}{16\pi G}\eta^{\alpha\beta\gamma\delta}(\omega_{\sigma\beta}\wedge\omega^\sigma{}_\gamma\wedge\theta_\delta - \omega_{\beta\gamma}\wedge\omega_{\sigma\delta}\wedge\theta^\sigma) \ , \tag{0.1}$$

where $\omega_{\alpha\beta}$ are the connection forms for the basis (θ^β) (Straumann 1985).

For our discussion we work within the weak field limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{0.2}$$

$(\eta_{\mu\nu}) = \mathrm{diag}(-1,1,1,1)$. We define

$$\overline{h}^{\mu\nu} \equiv h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h^\lambda_\lambda \ , \tag{0.3}$$

and adopt the harmonic gauge condition

$$\overline{h}^{\alpha\beta}{}_{,\beta} = 0 \ .$$

Einstein's field equations then take the well-known form

$$\Box\overline{h}_{\mu\nu} = -16\pi G T_{\mu\nu} + O(h^2) \tag{0.4}$$

with the retarded solution

$$\overline{h}_{\mu\nu}(\mathbf{x},t) = -4G\int\frac{T_{\mu\nu}(\mathbf{x}',t_{\mathrm{ret}})}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}' \ , \qquad t_{\mathrm{ret}} = t - |\mathbf{x}-\mathbf{x}'| \ . \tag{0.5}$$

Since $t_{\mathrm{ret}} = t - r + \mathbf{n}\cdot\mathbf{x}' + O(|x'|/r)|x'|$, we have

$$\overline{h}_{\mu\nu,i} = -\overline{h}_{\mu\nu,0}n_i + O(\overline{h}/r) \ ,$$

where $r = |\mathbf{x}|$ and $\mathbf{n} = \mathbf{x}/r$.
With the help of this identity and equation (0.1) one finds the energy, momentum and angular momentum radiated out to infinity by gravitational radiation:

$$< \dot{E} > \ = \ \frac{1}{32\pi G}\int_{\mathbf{S}} < h_{\alpha\beta,0}\overline{h}^{\alpha\beta,0} > r^2 d\Omega \ , \tag{0.6}$$

$$< \dot{P}^i > \ = \ \frac{1}{32\pi G}\int_{\mathbf{S}} < h_{\alpha\beta,0}\overline{h}^{\alpha\beta,0} > r^2 n^i d\Omega \ , \tag{0.7}$$

$$< \dot{L}^i > \ = \ -\frac{\epsilon^{ijk}}{16\pi G}\int_{\mathbf{S}} < 1/2\overline{h}_{\alpha\beta,k}h^{\alpha\beta}_{\ \ ,l} - \overline{h}^{\mu\rho}{}_{,k}\overline{h}_{\mu l,\rho} - \overline{h}_{k\mu,\rho}\overline{h}^{\mu\rho}{}_{,l} > n_j n_l r^3 d\Omega \ , \tag{0.8}$$

where $< \cdots >$ denotes the time average over a mean period of the radiation and $\mathbf{S}$ is a huge sphere of radius r containing the source. Equations (0.6) to (0.8) have been derived by Peters (1964) and Durrer (1989).

At first glance the integral on the r.h.s. of (0.8) seems to diverge in the limit $r \to \infty$, but one easily establishes that due to the antisymmetric tensor and the time average the $1/r^2$ terms in $< \cdots >$ vanish and $\dot{\mathbf{L}}$ remains finite. There is actually also an h^3 term which yields a finite contribution to $\dot{\mathbf{L}}$ in the limit $r \to \infty$, but for cosmic strings this term is always much smaller than the h^2 contribution (Durrer 1989). We therefore neglect it in the sequel.

2.2 Periodic Sources

Let us now consider a periodic source with period T:

$$T^{\mu\nu}(\mathbf{x}, t) = T^{\mu\nu}(\mathbf{x}, t+T) \ .$$

Setting $\omega_l = \frac{2\pi l}{T}$, $l \in \mathbf{N}$, we have

$$T^{\mu\nu}(\mathbf{x}, t) = \sum_{l=1}^{\infty} e^{-i\omega_l t} T^{\mu\nu}(\omega_l, \mathbf{x}) + \text{C.C.} \ . \tag{0.9}$$

(By 'C.C.' we denote the complex conjugate of the preceeding expression.)

The Fourier transform of $T^{\mu\nu}$ and its first moment can be given by

$$\begin{aligned} \mathcal{T}^{\mu\nu}(\omega_l, \mathbf{n}) &= \int d^3x T^{\mu\nu}(\omega_l, \mathbf{x}) e^{-i\mathbf{n}\cdot\mathbf{x}} \ , & (0.10) \\ \mathcal{T}^{\mu\nu p}(\omega_l, \mathbf{n}) &= \int d^3x T^{\mu\nu}(\omega_l, \mathbf{x}) x'^p e^{-i\mathbf{n}\cdot\mathbf{x}} \ . & (0.11) \end{aligned}$$

One harmonic mode of frequency ω in solution (0.5) expanded up to order $1/r^2$ inserted in the radiation formulas (0.6), (0.7) and (0.8) yields

$$\begin{aligned} < \frac{d\dot{E}(\omega)}{d\Omega} > &= -\frac{G\omega^2}{\pi} P_{ij} P_{lm} [\mathcal{T}^*_{il} \mathcal{T}_{jm} - \frac{1}{2} \mathcal{T}^*_{ij} \mathcal{T}_{lm}] \ , & (0.12) \\ < \frac{d\dot{P}_k(\omega)}{d\Omega} > &= -\frac{G\omega^2}{\pi} n_k P_{ij} P_{lm} [\mathcal{T}^*_{il} \mathcal{T}_{jm} - \frac{1}{2} \mathcal{T}^*_{ij} \mathcal{T}_{lm}] \ , & (0.13) \\ < \frac{d\dot{L}_i(\omega)}{d\Omega} > &= < \frac{d\dot{L}^{(1)}_i(\omega)}{d\Omega} > + < \frac{d\dot{L}^{(2)}_i(\omega)}{d\Omega} > \ , & (0.14) \end{aligned}$$

with

$$\begin{aligned} < \frac{d\dot{L}^{(1)}_i(\omega)}{d\Omega} > &= -\frac{iG\omega}{2\pi} \epsilon^{ijk} n^j n^l P^{pq} (3\mathcal{T}^*_{kl} \mathcal{T}^{qp} + 6\mathcal{T}^*_{kp} \mathcal{T}_{ql}) + \text{C.C.} & (0.15) \\ < \frac{d\dot{L}^{(2)}_i(\omega)}{d\Omega} > &= -\frac{G\omega^2}{2\pi} \epsilon^{ijk} n^j P^{lm} P^{pq} [2\mathcal{T}^*_{kmq} \mathcal{T}^{lp} - 2\mathcal{T}^*_{km} \mathcal{T}_{lpq} \\ &\quad -\mathcal{T}^*_{lpk} \mathcal{T}_{mq} + (1/2) \mathcal{T}^*_{lmk} \mathcal{T}_{pq}] + \text{C.C.} \ . & (0.16) \end{aligned}$$

$P_{ij} = \delta_{ij} - n_i n_j$ is the projection onto the plane orthogonal to $\mathbf{n}$.
Equation (0.12) is well-known, Weinberg (1972). Equation (0.14) is new. Its somewhat involved derivation is explicitly presented in Durrer (1989). The two terms $\dot{\mathbf{L}}^{(1)}$ and $\dot{\mathbf{L}}^{(2)}$ show in general a different asymptotic behaviour for $\omega \to \infty$ (see Section 3.2).

To find the total energy, momentum and angular momentum radiated one has of course to integrate over all directions and, for arbitrary periodic sources, to sum over all harmonic frequencies ω_l (cross terms vanish in the time average).

3 GRAVITATIONAL RADIATION OF COSMIC STRING LOOPS

3.1 Cosmic String Loops

Non–superconducting cosmic strings have no internal structure, i.e., no preferred frame of reference. Neglecting their finite thickness, they can thus be described by the Nambu action[2]. Parametrizing the points on a string world sheet by $(t, \mathbf{x}(\sigma, t))$, where σ is a spacelike parameter 'along the string' and denoting the derivative with respect to σ by a prime, the Nambu equations of motion in a flat background are given by

$$\ddot{\mathbf{x}} - \mathbf{x}'' = 0 \ , \tag{0.17}$$

if the gauge constraints

$$\dot{\mathbf{x}} \cdot \mathbf{x}' = 0 \ , \ \ \dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1 \quad \text{are satisfied.} \tag{0.18}$$

A general solution of (0.17) is of the form

$$\mathbf{x}(\sigma, t) = \frac{\mathrm{L}}{4\pi}[\mathbf{a}(\xi) + \mathbf{b}(\eta)] \ , \tag{0.19}$$

with $\xi = \frac{2\pi}{\mathrm{L}}(\sigma - t)$ and $\eta = \frac{2\pi}{\mathrm{L}}(\sigma + t)$. The constraints (0.18) yield

$$(\mathbf{a}')^2 = (\mathbf{b}')^2 = 1 \ ,$$

where here the prime denotes the derivative with respect to ξ and η respectively.

Let us now restrict our discussion to string loops, i.e. $\mathbf{a}$ and $\mathbf{b}$ have to be periodic. Choosing L such that $\mathbf{a}$ and $\mathbf{b}$ have the period 2π, we find

$$\mathbf{x}(\sigma + \mathrm{L}, t) = \mathbf{x}(\sigma, t) \ \text{ and therefore } \ \mathbf{x}(\sigma + \mathrm{L}/2, t + \mathrm{L}/2) = \mathbf{x}(\sigma, t) \ .$$

The fundamental period of a loop is thus $T = \mathrm{L}/2$.

In our gauge the energy momentum tensor of a loop is given by

$$T^{\mu\nu}(\mathbf{y}, t) = \mu \int_0^{\mathrm{L}} d\sigma (\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu) \delta^3(\mathbf{y} - \mathbf{x}(\sigma, t)) \ , \tag{0.20}$$

$(x^0(\sigma, t) = t.)$

Let us define the integrals

$$\begin{aligned}
I_k(l, \mathbf{n}) &\equiv \tfrac{1}{2\pi} \textstyle\int_0^{2\pi} d\xi e^{-il(\xi + \mathbf{n}\cdot\mathbf{a})} a'_k \\
J_k(l, \mathbf{n}) &\equiv \tfrac{1}{2\pi} \textstyle\int_0^{2\pi} d\eta e^{il(\eta - \mathbf{n}\cdot\mathbf{b})} b'_k \\
M_{kj}(l, \mathbf{n}) &\equiv \tfrac{1}{2\pi} \textstyle\int_0^{2\pi} d\eta e^{-il(\xi + \mathbf{n}\cdot\mathbf{a})} a'_k a_j \\
N_{kj}(l, \mathbf{n}) &\equiv \tfrac{1}{2\pi} \textstyle\int_0^{2\pi} d\eta e^{il(\eta - \mathbf{n}\cdot\mathbf{b})} b'_k b_j \ .
\end{aligned} \tag{0.21}$$

[2] It is clear to us that the two limits, thickness, $\delta \to 0$ and weak fields, are in principle not compatible. But we hope all the same that there exists a regime $(\delta, G\mu)$ where this limit yieldS, at least in order of magnitude, correct results. Our hopes have recently been nourished by investigations on exact solutions of cosmic string loops (Frolov, Israel & Unruh 1989) and by calculations of backreaction (Quashnock & Spergel 1988).

The Fourier transform of the energy momentum tensor (0.20) and its first moments can be expressed in terms of these integrals

$$\mathcal{T}_{kj}(\omega_l,\mathbf{n}) = -\frac{\mathrm{L}\mu}{2}[I_k(l,\mathbf{n})J_j(l,\mathbf{n}) + I_j(l,\mathbf{n})J_k(l,\mathbf{n})] \tag{0.22}$$

$$\mathcal{T}_{kij}(\omega_l,\mathbf{n}) = -\frac{\mathrm{L}^2\mu}{8\pi}[I_kN_{ij} + I_iN_{kj} + J_kM_{ij} + J_iM_{kj}]\ . \tag{0.23}$$

According to (0.12) to (0.16), the integrals (0.21) inserted in (0.22) and (0.23) yield the angular distribution of gravitational radiation of frequency ω_l for a given string loop (**a**,**b**). To obtain the total radiation emitted we have to integrate this distribution over all directions and to sum up over all frequencies $\omega_l,\ l \in \mathbf{N}$.

3.2 Asymptotics

The sum over all frequencies ω_l is of course not possible numerically. One has thus to find the asymptotic behaviour of $\dot{E}(\omega_l)$, $\dot{P}(\omega_l)$ and $\dot{L}(\omega_l)$ for large l. One can then calculate $\dot{E}(\omega_l)$, $\dot{P}(\omega_l)$ and $\dot{L}(\omega_l)$ numerically until the asymptotic regime is reached and estimate the remainder by the asymptotic behaviour.

To discuss the asymptotics, we define for arbitrary vectors **v**, **w**

$$\begin{aligned} I_l(\mathbf{n},\mathbf{v}) &= I_k(l,\mathbf{n})v^k & J_l(\mathbf{n},\mathbf{v}) &= J_k(l,\mathbf{n})v^k \\ M_l(\mathbf{n},\mathbf{v},\mathbf{w}) &= M_{kj}(l,\mathbf{n})v^kw^j & N_l(\mathbf{n},\mathbf{v},\mathbf{w}) &= N_{kj}(l,\mathbf{n})v^kw^j\ . \end{aligned} \tag{0.24}$$

Let us now choose $\mathbf{v},\mathbf{w}$ such that $(\mathbf{n},\mathbf{v},\mathbf{w})$ form an orthonormal frame. $\dot{E}(\omega,\mathbf{n})$, $\dot{\mathbf{P}}(\omega,\mathbf{n})$ and $\dot{\mathbf{L}}(\omega,\mathbf{n})$ can then be expressed in terms of the integrals (0.24) (see Durrer 1989).

In the first example of the next subsection (for $\alpha = 0$) the integrals (0.24) can be carried out analytically and yield Bessel functions. The asymptotic behaviour is thus most easily discussed and leads to the following results which are derived in Durrer (1989):

$$\dot{E}(\omega_l) \propto l^{-4/3}\ ,\quad \dot{P} = 0\ ,\quad \dot{L}^{(1)}(\omega_l) \propto l^{-2}\ ,\quad \dot{L}^{(2)}(\omega_l) \propto l^{-4/3}$$

for large l.
In the case of cuspless (but possibly kinky) loops ($\mathbf{a}(\xi) \neq -\mathbf{b}(\eta)\ \forall 0 \leq \xi,\eta \leq 2\pi$, Garfinkle & Vachaspati (1987) derived

$$I_l(\mathbf{n},\mathbf{v}) \propto \begin{cases} l^{-1}, & \text{if } \mathbf{n} \text{ does not lie on the } -\mathbf{a}' \text{ curve} \\ l^{-2/3}, & \text{if } \mathbf{n} \text{ lies on the } -\mathbf{a}' \text{ curve} \end{cases}$$

for all $\mathbf{v}$ orthogonal to $\mathbf{n}$. The same is true for $J_l(\mathbf{n},\mathbf{v})$ with $-\mathbf{a}'$ replaced by $\mathbf{b}'$. One easily obtains the same result for $M_l(\mathbf{n},\mathbf{v},\mathbf{w})$ and $N_l(\mathbf{n},\mathbf{v},\mathbf{w})$ for vectors $\mathbf{v}$,$\mathbf{w}$ orthogonal to $\mathbf{n}$. For $I_l(\mathbf{n},\mathbf{n})$ one finds by similar methods

$$I_l(\mathbf{n},\mathbf{n}) \propto \begin{cases} l^{-1}, & \text{if } \mathbf{n} \text{ does not lie on the } -\mathbf{a}' \text{ curve} \\ l^{-1/3}, & \text{if } \mathbf{n} \text{ lies on the } -\mathbf{a}' \text{ curve} \end{cases}$$

Again the same holds for $J_l(\mathbf{n},\mathbf{n})$ with $-\mathbf{a}'$ replaced by $\mathbf{b}'$.

For cuspless loops $\mathbf{n}$ can lie either on the $-\mathbf{a}'$ or on the $\mathbf{b}'$ curve but not on both since they do not intersect. Therefore either (I_l and M_l) or (J_l and N_l) can decay slower than l^{-1} but not both of them. Taking this into account one finds the asymptotic behaviour of $\mathcal{T}_{ij}(\omega_l,\mathbf{n})$ and $\mathcal{T}_{ijk}(\omega_l,\mathbf{n})$. Inserted in (0.12), (0.13) and (0.14) this yields the asymptotic behaviour for gravitational radiation of cuspless loops:

$$\dot{E}(\omega_l) \propto l^{-4/3}\ , \quad \dot{P} \propto l^{-4/3}\ , \quad \dot{L}^{(1)}(\omega_l) \propto l^{-2}\ , \quad \dot{L}^{(2)}(\omega_l) \propto l^{-4/3}$$

for large l. This is the same result as above, but it surely cannot be expanded to general 'cuspy' loops since there are well known examples where $\sum_l \dot{E}(\omega_l)$ and $\sum_l \dot{\mathbf{L}}(\omega_l)$ diverge (e.g. in our first numerical example for $\Phi = \pi$!).

3.3 Numerical Results

We have calculated the gravitational radiation numerically for the families of loops given below:

$$\begin{aligned}
\mathbf{a} &= [(1-\alpha)\sin\xi - (1/3)\alpha\sin 3\xi]\mathbf{e}_1 - [(1-\alpha)\cos\xi + (1/3)\alpha\cos 3\xi]\mathbf{e}_2 \\
&\quad + \sqrt{\alpha(1-\alpha)}(\sin 2\xi)\mathbf{e}_3\ , \qquad (0.25) \\
\mathbf{b} &= (\sin\eta)\mathbf{e}_1 - (\cos\Phi\cos\eta)\mathbf{e}_2 - (\sin\Phi\cos\eta)\mathbf{e}_3\ , \qquad (0.26) \\
&\quad 0 \le \alpha \le 1\ ;\ \ 0 \le \Phi \le \pi\ .
\end{aligned}$$

The energy and momentum radiation for some loops of this configuration have already been calculated by Vachaspati & Vilenkin (1985). For $\alpha \neq 0$ these loops are asymmetric, hence they radiate transversal momentum.

The angular momentum of a loop (0.25,0.26) is easily calculated with the result

$$\mathbf{L}^{(st)} = -\frac{\mu \mathrm{L}^2}{4\pi}[\sin(\Phi/2)\{\cos(\Phi/2)\mathbf{e}_2 + \sin(\Phi/2)\mathbf{e}_3\} + \alpha(\alpha/3-1)\mathbf{e}_3]\ . \qquad (0.27)$$

These are all loops with cusps. As a second example we calculated the radiation for a family of kinky but cuspless loops:

$$\begin{aligned}
\mathbf{a} &= (1/p)\sin(p\xi+\beta)\mathbf{e}_1 - (1/p)\cos(p\xi+\beta)\mathbf{e}_2\ , \\
&\quad \beta = (1-p)\pi[\xi/\pi]\ , \qquad (0.28) \\
\mathbf{b} &= (1/q)\sin(q\eta+\delta)\mathbf{e}_1 - (1/q)\cos(q\eta+\delta)(\cos\Phi\mathbf{e}_2 + \sin\Phi\mathbf{e}_3)\ , \\
&\quad \delta = (1-q)(\pi/2 + \pi[\eta/\pi])\ , \qquad (0.29)
\end{aligned}$$

$0 < p, q < 1$ (Garfinkle & Vachaspati 1987).
The square bracket $[x]$ denotes the nearest lower integer.
For these loop configurations the angular momentum is given by

$$\mathbf{L}^{(st)} = -\frac{\mu \mathrm{L}^2}{8\pi}[\frac{\sin\Phi}{q}\mathbf{e}_2 + (\frac{1}{p} - \frac{\cos\Phi}{q})\mathbf{e}_3]\ . \qquad (0.30)$$

Our numerical results are presented in Figs 1 to 7 and Tables 1 to 4. In Figs. 1 to 4, we plot $\dot{E}(\omega_N)$, $\dot{P}(\omega_N)$ and $\dot{L}(\omega_N)$ for some configurations. In Figs. 2 and 3 the actual falloff of $\dot{L}$ is compared with the asymptotic behaviour calculated in the previous subsection. In Fig. 4 one sees that for each mode of radiation $\dot{\mathbf{L}}(\omega_N)$ is perfectly antiparallel to $\mathbf{L}^{(\mathbf{st})}$. This remarkable result holds also for all other examples which we have investigated (see Tables 1 and 3). Angular momentum radiation tends thus always to diminish the angular momentum of the loop and does never rotate or even increase it! Unfortunately we are (up to now) not able to give a general proof of this purely numerical result. In Durrer (1989) we have shown that for $\mathbf{a} \equiv \mathbf{b}$, $\dot{\mathbf{L}} = 0$. Since these are the only C^∞ – loops with $L^{(st)} = 0$, this means that for $L^{(st)} = 0$ also $\dot{\mathbf{L}} = 0$. Loop configurations (0.25), (0.26) with $\alpha = 0$ exhibit no radiation of transversal momentum. In Figs. 5, 6 and 7 energy and angular momentum radiation of these loops are given as functions of the parameter Φ. $\dot{E}$ diverges for $\Phi = 0, \pi$ and $\dot{L}$ diverges for $\Phi = \pi$ (see Vachaspati & Vilenkin 1985, Durrer 1989). The numerical result is compared with the result obtained in quadrupole approximation. To our surprise the quadrupole approximation is never more than a factor of 2 off from the numerical value. We suppose that this is due to the high degree of symmetry of $\alpha = 0$ loops, since for $\alpha \neq 0$ the corresponding difference can amount to up to two orders of magnitude (see Table 2).

According to Fig. 7 and Tables 3 and 4 we adopt the following 'mean' values of $\dot{E}$, $\dot{P}$ and $\dot{\mathbf{L}}$: $\dot{E} = \gamma_E G\mu^2$, $\dot{P} = \gamma_P G\mu^2$, $\dot{\mathbf{L}} = -\gamma_L G\mu^2 \mathrm{L}\mathbf{e}^{(st)}$ ($\mathbf{e}^{(st)}$ is the direction of $\mathbf{L}^{(st)}$) with

$$\begin{aligned}
\gamma_E &\simeq 50 , \\
\gamma_P &\simeq 5 , \\
\gamma_L &\simeq 5 .
\end{aligned} \tag{0.31}$$

The direction of $\dot{\mathbf{L}}$ seems not to be correlated with the direction of $\dot{\mathbf{P}}$.

$(\Phi/\pi, \alpha)$	$\frac{<\dot{\mathbf{L}}\cdot\mathbf{L}^{(st)}>}{\lvert\dot{\mathbf{L}}\rvert\cdot\lvert\mathbf{L}^{(st)}\rvert}$	$\frac{<\dot{\mathbf{L}}\cdot\dot{\mathbf{P}}>}{\lvert\dot{\mathbf{L}}\rvert\cdot\lvert\dot{\mathbf{P}}\rvert}$
(0.0 , 0.5)	-1.0	0.78
(0.25, 0.5)	-0.98	-0.4
(0.5 , 0.5)	-1.0	-0.93
(0.75, 0.5)	-0.99	-0.37
(0.25, 0.8)	-0.99	-0.84
(0.5 , 0.8)	-1.0	-0.48
(0.75, 0.8)	-0.98	0.36

Table 1: The angle between the angular momentum radiated away, $\dot{\mathbf{L}}$, and $\mathbf{L}^{(st)}$ respectively $\dot{\mathbf{P}}$.

$(\Phi/\pi, \alpha)$	$\dot{E}$	$\dot{E}^{(q)}$	$\dot{\mathbf{L}}$	$\dot{\mathbf{L}}^{(1)}$	$\dot{\mathbf{L}}^{(2)}$	$\dot{\mathbf{L}}^{(q)}$
(0.0 , 0.5)	52.3	2.63	3.67	2.55	1.12	0.0
(0.25, 0.5)	54.1	3.16	3.45	2.29	1.16	2.21
(0.5 , 0.5)	56.9	5.59	4.07	2.43	1.65	5.33
(0.75, 0.5)	56.6	9.61	4.81	2.92	1.91	8.61
(0.25, 0.8)	75.1	0.51	6.09	3.61	2.48	0.35
(0.5 , 0.8)	63.9	0.89	4.46	2.43	2.08	0.85
(0.75, 0.8)	48.1	1.55	3.02	1.45	1.63	1.38

Table 2: Comparison of the numerical values of energy and angular momentum radiation with the values predicted by the quadrupole approximation. ($\dot{E}$ in units $G\mu^2$, $\dot{L}$ in units $G\mu^2$Ł.)

$\Phi/\pi, (p, q)$	γ_E	γ_L	$\frac{<\dot{\mathbf{L}}\cdot\mathbf{L}^{(st)}>}{\lvert\dot{\mathbf{L}}\rvert\cdot\lvert\mathbf{L}^{(st)}\rvert}$
0.8 , (0.4,0.2)	22.	7.4	0.99
0.5, (0.6,0.4)	19	2	-0.99
0.5, (0.4,0.8)	26	4	-0.97
0.5, (0.9,0.9)	42	4.5	-1.0

Table 3: Energy and angular momentum radiation for some loops of the class (0.28), (0.29).

Figure 1: Energy , $-\dot{E}$ ($\diamond$), and momentum radiation, $|\dot{\mathbf{P}}|$ ($\circ$), are drawn logarithmically as a function of the mode number N, $\omega_N = 4\pi N/\text{Ł}$, in units of $G\mu^2$. For different values of the parameters $(\Phi, \alpha \neq 0)$ the corresponding plots look similar.

4 COSMOLOGICAL CONSEQUENCES

In this section we discuss the implications of our results to cosmic string loops in a flat, matter dominated Friedman universe. (The scale factor evolves thus according to $a \propto t^{2/3}$.) As long as the length of the loop is much smaller than the horizon size ($\mathrm{L} \ll t$) the flat spacetime results of the preceeding section remain valid.

4.1 The Rocket–Effect

Hogan & Rees (1984) suggested that due to asymmetrical gravitational radiation cosmic string loops might speed up and travel through the universe at extremely high speed.

For a loop of length L, born (i.e. chopped off an infinite string, see AT and BB) at time t_i with constant radiation rates $\dot{P} = \gamma_P G\mu^2$ and $\dot{E} = \gamma_E G\mu^2$ and initial momentum p_i, we find

$$v(t) = \frac{p_i x^{-2/3} + (3/5)\gamma_P G\mu^2 t_i x}{[(\mu\mathrm{L} - (x-1)(\gamma_E G\mu^2 t_i))^2 + (p_i x^{-2/3} + (3/5)\gamma_P G\mu^2 t_i x)^2]^{1/2}} ,$$

where $x = t/t_i$. For $v_i \approx 0.5$, $\gamma_P \approx 2$ and $t_i \approx \mathrm{L}$ the result simplifies to

$$v(t) \simeq [\frac{(x^{-2/3} + G\mu x)^2}{(x^{-2/3} + G\mu x)^2 + (1 - G\mu\gamma_E x)^2}]^{(1/2)} . \tag{0.32}$$

For this case $v(t)$ is shown in Fig. 8. For reasonable (i.e. relativistic) initial velocities $v(t) \geq v_{min} \approx 0.01$, and dynamical friction never becomes important (Durrer 1989). The velocity then really evolves according to equation (32).

We thus conclude that cosmic string loops typically have highly relativistic peculiar velocities. As we discuss in the next subsection, this result substantially alters the picture of accretion of matter onto string loops.

Of course all the results presented in this section have to be taken with a grain of salt. First of all, we do not know how the mean values γ_E, γ_P and γ_L of our numerical examples are related to corresponding values of a cosmological network. We just hope that they are of the same order of magnitude. (For some justification of this hope see Durrer (1989).) Furthermore, the radiation backreaction was taken into account only by the conservation laws, so that it alters the string energy, momentum and angular momentum but not its shape. This is of course far from obvious and it remains an important open question for how many oscillations $\dot{\mathbf{P}}$ points approximately in the same direction. If this number of coherent oscillations, N_c, is big, $N_c \sim 1000$, our results remain qualitatively valid. But if $N_c \sim 10$, the acceleration of the loops has to be treated as a random walk and the velocity is substantially reduced. The fact that gravitational radiation does not change the direction of $\mathbf{L}^{(\mathbf{st})}$ and recent investigations on the backreaction problem of Quashnock & Spergel (1989) hint that N_c might in fact be rather large.

Figure 8: The velocity evolution of a cosmic string loop born at relativistic speed, $v_i \simeq 0.5$, with $\mathrm{L} \simeq t_i$ is shown. The decay time, $t_d \simeq 2 \cdot 10^4 t_i$ is indicated

4.2 Accretion of Matter around Fast Moving Cosmic String Loops

As before, we denote by t_i the time when the loop was born. Let us first neglect the finite size of a cosmic string loop. We can then apply a simple model proposed by Bertschinger (1987) for accretion of cosmic dust onto a fast moving point source. We consider a disk orthogonal to the velocity of the loop with the position of the loop at a time t_0 at its center. Within this model, the radius of the disk which has been accreted (i.e. turned around) until some time t is given by

$$d_{acc}(t_0, t) \simeq t_i (t/t_i)^{1/3} (\tfrac{G\mu \mathrm{L}}{v_{st} t_0})^{1/2} <$$

$$d_{acc}(t_i, t_d) \simeq t_i (\mathrm{L}/t_i)^{5/6} (\gamma_E G\mu)^{-1/3} (G\mu/v_{st})^{1/2} \simeq 0.1 \mathrm{L} (t_i/\mathrm{L})^{1/6},$$

where t_d denotes the decay time of the loop and v_{st} is a typical string velocity. The last approximation was obtained setting $v_{st} = 0.1$, $\gamma_E = 50$ and $G\mu = 10^{-6}$. This result shows that the finite size of fast moving loops cannot be neglected.

On the other hand, treating the string as infinite, $d_{acc} \ll \mathrm{L}$, yields (Bertschinger, 1987) $d_{acc} \simeq 40 \mathrm{L} (\mathrm{L}/t_i)^{1/3}$, which again means that this approximation is not valid. Hence, for reasonable values of $1 > \mathrm{L}/t_i > 10^{-6}$, we conclude that the radius of the accreted matter is comparable with the size of the loop,

$$d_{acc}(t_i, t_d) \simeq \mathrm{L} \ .$$

(Note that the lifetime of a loop is $(\gamma_E G\mu)^{-1} \mathrm{L} \sim 2 \cdot 10^4 \mathrm{L}$. Thus if $\mathrm{L}/t_i < 0.5 \cdot 10^{-4}$, the loop lives for less than one expansion time and thus has not much time to accrete matter.)

The above result is valid only for cold particles. We have neglected thermal velocities. For particles with some finite mean thermal velocity v_T, d_{acc} has to be compared with a typical thermal distance

$$d_T \sim v_T(t_d - t_i) \ .$$

For accretion to take place at all, we must require $d_T < d_{acc}$, i.e.,

$$v_T < \mathrm{L}/(t_d - t_i) \sim 10^{-4} \ .$$

In particular, hot dark matter cannot be accreted onto fast moving loops.

The results of this section together with observational limits imposed on $G\mu$ from bounds on the gravitational wave background (see AT and BB) imply that cosmic string loops are most probably unimportant for structure formation in the universe. Whether or not infinite strings can produce the observed structure remains unclear (see contributions of Brandenberger and Stebbins in this proceedings).

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Figure 2: Angular momentum radiation ($\dot{L}_2^{(1)}(\circ)$, $\dot{L}_3^{(1)}(\bullet)$, $\dot{L}_2^{(2)}(\triangle)$ and $\dot{L}_3^{(2)}(\diamondsuit)$ is drawn logarithmically as a function of the mode number N in units of Ł$G\mu^2$. The numerical falloff is compared with the theoretically predicted one.

Figure 3: The same as Figure 2 for a kinky loop configuration.

Figure 4: $|\dot{\mathbf{L}}|$ in units of (Ł$G\mu^2$) ($\bullet$) and $\frac{\dot{\mathbf{L}}\cdot\mathbf{L}^{(st)}}{|\dot{\mathbf{L}}||\mathbf{L}^{(st)}|}$ ($\star$, linear scale) are drawn as functions of the mode number. The same diagram for different loop configurations looks quite similar.

Figure 5: Energy radiation for $\alpha = 0$ configurations ($\diamondsuit$) is shown as a function of the parameter Φ and is compared with the amount expected in quadrupole approximation ($\triangle$). The units are $G\mu^2$.

Figure 6: The total angular momentum radiation ($\triangle$), for $\alpha = 0$ configurations is shown as a function of the parameter Φ in units $\mathrm{L}G\mu^2$ and is compared with the amount expected in quadrupole approximation, $\dot{\mathbf{L}}^{(q)}$ ($\diamondsuit$). ($|\dot{\mathbf{L}}^{(1)}|$ ($\circ$), $|\dot{\mathbf{L}}^{(2)}|$ ($\bullet$))

Figure 7: For $\alpha = 0$ configurations the values γ_E ($\diamondsuit$) and γ_L ($\triangle$) are drawn as functions of the parameter Φ.

$(\Phi/\pi, \alpha)$	γ_E	γ_P	γ_L
(0.0 , 0.5)	52.3	0.02	3.67
(0.25, 0.5)	54.1	5.66	3.45
(0.5 , 0.5)	56.9	2.35	4.07
(0.75, 0.5)	56.6	5.51	4.81
(0.25, 0.8)	75.1	2.31	6.09
(0.5 , 0.8)	63.9	1.79	4.46
(0.75, 0.8)	48.1	0.84	3.02

Table 4: Energy, momentum and angular momentum radiation for some loops of the class (0.25), (0.26).