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Gravitational waves from self-ordering scalar fields

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Abstract: Gravitational waves were copiously produced in the early Universe whenever the processes taking place were sufficiently violent. The spectra of several of these gravitational wave backgrounds on subhorizon scales have been extensively studied in the literature. In this paper we analyze the shape and amplitude of the gravitational wave spectrum on scales which are superhorizon at the time of production. Such gravitational waves are expected from the self ordering of randomly oriented scalar fields which can be present during a thermal phase transition or during preheating after hybrid inflation. We find that, if the gravitational wave source acts only during a small fraction of the Hubble time, the gravitational wave spectrum at frequencies lower than the expansion rate at the time of production behaves as $\Omega_{\rm GW}(f) \propto f^3$ with an amplitude much too small to be observable by gravitational wave observatories like LIGO, LISA or BBO. On the other hand, if the source is active for a much longer time, until a given mode which is initially superhorizon ($k\eta_* \ll 1$), enters the horizon, for $k\eta \gtrsim 1$, we find that the gravitational wave energy density is frequency independent, i.e. scale invariant. Moreover, its amplitude for a GUT scale scenario turns out to be within the range and sensitivity of BBO and marginally detectable by LIGO and LISA. This new gravitational wave background can compete with the one generated during inflation, and distinguishing both may require extra information.

Keywords: Gravitational wave background, inflationary cosmology, reheating the Universe, thermal phase transitions.

Contents

1. Introduction

Gravitational waves (GWs) are produced in the late Universe via cataclismic astrophysical events like hypernovae and inspiralling binaries. Because gravity is so weak, it is extremely difficult to detect directly with present day interferometers [1]. On the other hand, during the violent processes which we expect took place in the very early Universe, several stochastic backgrounds of GWs of significant energy may be produced, although their amplitude today is drastically reduced by the expansion of the Universe, making them equally difficult to detect [2]. Their discovery may however be possible in the near future, opening a completely new window into the uncharted territory of the very early Universe. For this we must determine the detailed GW spectrum, which strongly depends on the physical processes generating them.

In the last few years there has been significant progress in the experimental prospects for detecting GWs with interferometers like LIGO and VIRGO and the future satellite mission LISA. This has stimulated research for sources of primordial GWs from the early Universe, either from hypothetical first order phase transitions [3, 4, 5, 6, 7, 8] or from the process of reheating after inflation [9, 10, 11, 12, 13, 14, 15, 16].

The mechanism responsible for GW production during these early Universe phenomena is typically a causal process, like bubble collisions or turbulence, giving rise to spectra which peak at wavelengths that are well within the causal horizon during their generation. Thus, most of past analyses concentrate on contributions of GWs with wavelengths smaller than the horizon at the time of production, with the exception of those generated during inflation [17], which are stretched by the inflationary expansion.

In this paper we study the infrared behaviour of the GW spectrum produced either during preheating or during first order phase transitions, on scales which are superhorizon at the time of formation, *i.e.* $k < \mathcal{H}_*$, where k and $\mathcal{H}_*$ are the comoving momentum and inverse horizon. We want to study a causal process of symmetry breaking like hybrid preheating [18, 19, 20, 21, 22, 23], where the order parameter has global $\mathcal{O}(N)$ symmetry in the false vacuum and, upon symmetry breaking, the N fields undergo self-ordering on a given scale as soon as they enter the horizon, in particular on scales much larger than the inverse mass of the field in the true vacuum.

We consider a multi-component scalar field which obtains a non-zero *vacuum expectation value* (*vev*) v and a mass m, during a symmetry breaking process. We shall assume that this mass m is much larger than the Hubble parameter H_* at the time of the transition, since if the *vev* in the true vacuum is much smaller than the Planck scale, then $H_* \sim m\,v/M_p \ll m$. Such a model could describe the symmetry breaking process which triggers the end of hybrid inflation or a thermal phase transition. As long as we are only interested in superhorizon scales, $k \gg \mathcal{H}_*$, we can neglect the radial, massive mode and treat the dynamics within the non-linear sigma-model (NLSM) approximation. On large scales, the anisotropic stresses are determined by gradient energy and the typical (comoving) scale is simply the time dependent horizon scale $\mathcal{H}^{-1}$. The field self-orders at the horizon scale, and the source of GWs decays inside the horizon. For scalar metric perturbations this process has been studied *e.g.* in Ref. [24]. It is very closely related to the scaling of global topological defects [25] even though for a number of components $N > 4$ there are no topological defects associated with such a scalar field in $3 + 1$ dimensions.

We work in the large N approximation within which the scalar field equation of motion, for scales larger than the inverse mass, $k \ll m$, can be solved analytically. The GW spectrum will then be estimated by analytical approximations, introducing the anisotropic stress tensor sourced by the field fluctuations at different scales.

Tensor perturbations from a NLSM in the large N approximation have also been studied in Ref. [26, 27], see also [28]. There the authors have calculated the tensor perturbation spectrum for scales which enter the horizon in the matter era and they have compared this with the inflationary signal in the CMB. Here we shall concentrate on the radiation dominated era and the detection of the signal in direct gravitational wave experiments like advanced LIGO [29], LISA [30] and BBO [31].

The paper is organized as follows. In the next section we describe the formalism, derive the scalar field solutions and calculate the unequal time anisotropic stress correlators which source GWs. In Section 3 we study the production of GWs from long wavelength modes of this source. We derive a general formula that can be applied to different situations, depending how long the GW source is acting. In Section 4 we use this result to determine the shape and amplitude of the GW spectrum in two situations, first the case of a source producing GWs only during a small fraction of the Hubble time and, second, the case in which the source producing GWs acts for a much longer time, until a given mode which is initially superhorizon, $k\eta_* \ll 1$, enters the Hubble radius, $k\eta \simeq 1$. In Section 5 we summarize our results and conclude.

Notation Throughout this paper we assume a spatially flat Friedmann Universe with metric

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \delta_{ij}dx^i dx^j\right) , \tag{1.1}$$

where η denotes conformal time and we normalize the scale factor to unity today, $a(\eta_0) = 1$. The comoving Hubble rate is $\mathcal{H} = a'/a$, while $H = a'/a^2$ is the physical one. The prime denotes derivative w.r.t. conformal time η.

2. Formalism

We first introduce the NLSM and the large N limit of a global $\mathcal{O}(N)$ symmetric scalar field, then we study the physics of the correlators of the anisotropic stress tensor.

2.1 The model

We consider an N-component scalar field with a Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 = -\partial_\mu \Phi^{\rm T}\partial^\mu\Phi - \lambda\left(\Phi^{\rm T}\Phi - \frac{v^2}{2}\right)^2 + \mathcal{L}_1 , \tag{2.1}$$

where $\Phi^{\rm T} = (\phi_1, \phi_2, ..., \phi_N)/\sqrt{2}$, λ is the dimensionless self-coupling of Φ and v is the *vev* in the true vacuum. In the case of a thermal bath at high temperature, the Lagrangian $\mathcal{L}_0$ obtains corrections of the form $\mathcal{L}_1 \sim -T^2\Phi^2$, so that its minimum is at $\Phi = 0$ which respects the global $\mathcal{O}(N)$ symmetry of the Lagrangian. At low temperature, $T < T_c \simeq v$, the thermal corrections are too small to the keep the minimum at $\Phi = 0$ and the global $\mathcal{O}(N)$ symmetry is spontaneously broken to $\mathcal{O}(N-1)$. In the context of hybrid preheating, there is no need for thermal restoration of the symmetry. The field Φ acquires a large mass during inflation through its coupling to the inflaton χ, $\mathcal{L}_1 = -g^2\Phi^{\rm T}\Phi\chi^2$. Above a critical value, $\chi > \chi_c \equiv \sqrt{\lambda}v/g$, the effective quadratic mass of Φ is positive and the field is fixed at $\Phi = 0$. When the quadratic mass becomes negative, $\chi < \chi_c$, a tachyonic instability triggers the end of inflation and symmetry breaking. Soon after the symmetry is broken, thermal corrections and tachyonic effects can be neglected, and Φ is closely confined (in most of space) to the vacuum manifold, given by $\sum_a \phi_a^2(\mathbf{x}, \eta) = v^2$. Nevertheless, in positions such that their comoving distance is $|\mathbf{x} - \mathbf{x}'| > \mathcal{H}^{-1}$, the values $\Phi(\mathbf{x}, \eta)$ and $\Phi(\mathbf{x}', \eta)$ are uncorrelated, which leads to a gradient energy density associated to the $N - 1$ Goldstone modes, $\rho \sim (\partial_i\Phi)^2$. For $N > 2$, the dynamics of the Goldstone modes is well described by a NLSM [32, 25] where we force $\sum_a \phi_a^2 = v^2$ by a Lagrange multiplier. This corresponds to the limit $\lambda \to \infty$ in the above Lagrangian. This approximation is very good for physical scales with are much larger than $m^{-1} \equiv 1/(\sqrt{\lambda}v)$. Of course, on small scales the field fluctuations still oscillates around the true *vev*, but in this paper we only focus on the superhorizon modes which are free to wander around in the vacuum manifold, giving rise to a gradient energy density which will generate GWs on these scales.

Normalizing the symmetry breaking field to its *vev*, $\beta \equiv \Phi/v$, each component of the field obeys the non-linear sigma model evolution equation [24]

$$\Box\beta^a - (\partial_\mu\beta \cdot \partial^\mu\beta)\beta^a = 0 , \tag{2.2}$$

where $(\partial_\mu\beta \cdot \partial^\mu\beta) = \sum_a \eta^{\mu\nu}\partial_\mu\beta^a(\mathbf{x}, \eta)\partial_\nu\beta^a(\mathbf{x}, \eta)$ and $\sum_a \beta^a(\mathbf{x}, \eta)\beta^a(\mathbf{x}, \eta) = 1$. In the large N-limit, we assume that the sum over components can be replaced by an ensemble average,

$$T(x) = \sum_a \eta^{\mu\nu}\partial_\mu\beta^a\partial_\nu\beta^a = N\langle\eta^{\mu\nu}\partial_\mu\beta^a\partial_\nu\beta^a\rangle = \bar{T}(\eta) . \tag{2.3}$$

By dimensional considerations, $T \propto \mathcal{H}^2$, or

$$\bar{T}(\eta) = T_o \eta^{-2} \,, \tag{2.4}$$

with $T_o > 0$. Replacing the non-linearity in the sigma-model by this expectation value we obtain a linear equation which can be solved exactly. In Fourier space it reads

$$\beta_k^{a\,\prime\prime} + \frac{2\gamma}{\eta}\beta_k^{a\,\prime} + \left(k^2 - \frac{T_o}{\eta^2}\right)\beta_k^a = 0 \,, \tag{2.5}$$

where $\gamma = d\log a/d\log\eta$ and primes denote derivatives w.r.t. η. In a radiation dominated Universe $\gamma = 1$ while in a matter dominated Universe $\gamma = 2$. The solution to Eq. (2.5) for constant γ is given by

$$\beta^a(\mathbf{k},\eta) = (k\eta)^{\frac{1}{2}-\gamma}\Big[C_1(\mathbf{k})\, J_\nu(k\eta) + C_2(\mathbf{k})\, Y_\nu(k\eta)\Big] \,, \tag{2.6}$$

where

$$\nu^2 = \left(\frac{1}{2} - \gamma\right)^2 + T_o \,, \tag{2.7}$$

and C_1, C_2 are constants of integration. Thus, $\nu > 1/2$ for a radiation dominated Universe and $\nu > 3/2$ for matter domination. Since in general we have that $\nu > 0$, Y_ν diverges for small argument, so we will keep only the regular mode of the solution J_ν, which can be written as

$$\beta^a(\mathbf{k},\eta) \;=\; \sqrt{A}\left(\frac{\eta}{\eta_*}\right)^{\frac{1}{2}-\gamma} \frac{J_\nu(k\eta)}{(k\eta_*)^\nu}\beta^a(\mathbf{k},\eta_*) \,, \tag{2.8}$$

where $\beta^a(k,\eta_*)$ is the a-th component of the field at the initial time η_*. We assume that β is initially Gaussian distributed with a scale-invariant spectrum on large scales and vanishing power on small scales

$$\langle \beta^a(\mathbf{k},\eta_*)\beta^{*b}(\mathbf{k}',\eta_*)\rangle = \left\{ \begin{array}{cl} (2\pi)^3\mathcal{C}\frac{\delta^{ab}}{N}\delta(\mathbf{k}-\mathbf{k}') & ,\ k\eta_* \ll 1 \\ 0 & ,\ k\eta_* > 1 \,. \end{array} \right. \tag{2.9}$$

This means that the field is aligned on scales smaller than the comoving horizon η_* and has arbitrary orientation on scales larger than η_*. The condition that $\beta^2 = 1$ actually introduces correlations between the different components of β but these lead to corrections of order $1/N$ to the above expression which we will neglect here. We also do not enter into the details of the decay of this function around $k\eta_* = 1$. The constant $\mathcal{C}$ is chosen such that the normalization condition is satisfied (up to corrections of order $1/N$),

$$\begin{aligned} \beta^2(\mathbf{x},\eta_*) &\equiv \langle \beta^2(\mathbf{x},\eta_*)\rangle\Big(1 + \mathcal{O}(1/N)\Big) \\ &\simeq \int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\langle \beta^a(\mathbf{k},\eta_*)\beta^{*a}(\mathbf{k}',\eta_*)\rangle e^{i\mathbf{x}\cdot(\mathbf{k}-\mathbf{k}')} \simeq \frac{\mathcal{C}}{6\pi^2\eta_*^3} = 1 \,. \end{aligned} \tag{2.10}$$

In the large N-limit we neglect the corrections of order $1/N$ which come from the fluctuations in β^2. On large scales this is a very good approximation. However, on small scales, and in particular, on scales comparable with the inverse of the mass of the symmetry breaking field, m^{-1}, the fluctuations are certainly not negligible. In our analysis we consider only large scales, where the above approximation is valid.

In order for $\langle\beta^2\rangle$ to be time independent we need that the equal time correlator be fixed to one:

$$\langle\beta^2(\mathbf{k},\eta)\rangle = A\,\mathcal{C}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\eta}{\eta_*}\right)^{(1-2\gamma)}\frac{J_\nu^2(k\eta)}{(k\eta_*)^{2\nu}}$$
$$\simeq 3A\left(\frac{\eta_*}{\eta}\right)^{2(1+\gamma-\nu)}\int_0^\infty dy y^{2(1-\nu)}J_\nu^2(y) = 1\,, \tag{2.11}$$

where we have substituted $\mathcal{C}=6\pi^2\eta_*^3$ and we have set $y=k\eta$. Note that the upper limit is actually η/η_*, but at late times, the (dimensionless) integral is insensitive to the upper boundary, so we can take it to infinity and thus make the integral free of any time scale. In order to obtain a time-independent *vev*, we then just require

$$\nu=\gamma+1\ . \tag{2.12}$$

Introducing this relation into Eq. (2.7), one obtains T_o in terms of γ as

$$T_o = 3(\gamma+1/4)\ . \tag{2.13}$$

The constant A is determined then by the condition

$$1=3A\int_0^\infty dy y^{2(1-\nu)}J_\nu^2(y)\ ,\qquad\text{hence}\qquad A=\frac{4\Gamma(2\nu-1/2)\Gamma(\nu-1/2)}{3\Gamma(\nu-1)}\ . \tag{2.14}$$

Since $\nu=\gamma+1$, we can also write the amplitude of the field fluctuations, as

$$\beta^a(\mathbf{k},\eta)=\sqrt{A}\left(\frac{\eta}{\eta_*}\right)^{3/2}\frac{J_\nu(k\eta)}{(k\eta)^\nu}\beta^a(\mathbf{k},\eta_*)\ . \tag{2.15}$$

2.2 Unequal time correlators

From Eqs. (2.9) and (2.15) we obtain the following expression for the unequal time correlator of the field:

$$\left\langle\beta^a(\mathbf{k},\eta)\beta^{*b}(\mathbf{k}',\eta')\right\rangle = A\left(\frac{\eta\eta'}{\eta_*^2}\right)^{3/2}\frac{J_\nu(k\eta)J_\nu(k'\eta')}{(k\eta)^\nu(k'\eta')^\nu}\left\langle\beta^a(\mathbf{k},\eta_*)\beta^{*b}(\mathbf{k}',\eta_*)\right\rangle$$
$$=(2\pi)^3 6\pi^2 A(\eta\eta')^{3/2}\frac{J_\nu(k\eta)J_\nu(k\eta')}{(k\eta)^\nu(k\eta')^\nu}\frac{\delta_{ab}}{N}\delta(\mathbf{k}-\mathbf{k}')$$
$$\equiv(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\mathcal{P}_\beta^{ab}(k,\eta,\eta')\,. \tag{2.16}$$

We assume that the field β is Gaussian distributed initially. As its time evolution is linear, it will remain a Gaussian field and we can determine higher order correlators via Wick's theorem. This will be important in the next section when we determine the unequal time correlator of the anisotropic stresses which source the production of GWs.

Furthermore, this source is *totally coherent* [25] in the sense that its unequal time correlator $\mathcal{P}_\beta^{ab}(k,\eta,\eta')$ is a product of a function of η and η',

$$\mathcal{P}_\beta^{ab}(k,\eta,\eta') = \frac{\delta_{ab}}{N} 6\pi^2 A(\eta\eta')^{3/2} \frac{J_\nu(k\eta)J_\nu(k\eta')}{(k\eta)^\nu(k\eta')^\nu} \equiv \frac{\delta_{ab}}{N} f(k,\eta)f(k,\eta') \ , \tag{2.17}$$
$$\text{with} \quad f(k,\eta) = \sqrt{6\pi^2 A}\ k^{3/2} \frac{J_\nu(k\eta)}{(k\eta)^{\nu-3/2}} \ .$$

Note the $k^{3/2}$ scaling law at horizon crossing ($k\eta \sim 1$) which is characteristic for quantum fluctuations from de Sitter, *i.e.* inflation. This already hints to the fact that we will find a scale-invariant spectrum also in this case.

3. The production of gravitational waves

In this section we derive a general formula for the GW power spectrum sourced by super-horizon modes of a self ordering field. We also comment about the frequency range for the GW background produced in this way.

Let us consider tensor perturbations (GWs) of the metric,

$$ds^2 = a^2(\eta)(\eta_{\mu\nu} + 2h_{\mu\nu})dx^\mu dx^\nu \ , \tag{3.1}$$

where h_{ij} is traceless, $h^i_i = 0$, and divergence free, $\partial^i h_{ij} = 0$. Linearizing Einstein's equations yields the evolution equation of GWs sourced by the anisotropic stresses of the scalar fields Φ,

$$h''_{ij}(\mathbf{x},\eta) + 2\mathcal{H}\, h'_{ij}(\mathbf{x},\eta) - \nabla^2 h_{ij}(\mathbf{x},\eta) = 8\pi G\, \Pi_{ij}(\mathbf{x},\eta) \ , \tag{3.2}$$

where Π_{ij} represents the `TT` part of the (effective) anisotropic stress tensor

$$T_{ij}(\mathbf{x},\eta) = \partial_i\phi^a(\mathbf{x},\eta)\partial_j\phi^a(\mathbf{x},\eta) - \frac{1}{3}\delta_{ij}\left[\nabla\phi^a(\mathbf{x},\eta)\right]^2 \ . \tag{3.3}$$

Fourier transforming the GW evolution equation (3.2) we obtain

$$h''_{ij}(\mathbf{k},\eta) + 2\mathcal{H}\, h'_{ij}(\mathbf{k},\eta) + k^2 h_{ij}(\mathbf{k},\eta) = 8\pi G\, \Lambda_{ij,lm}(\hat{\mathbf{k}})T_{lm}(\mathbf{k},\eta) \tag{3.4}$$

where the projector

$$\Lambda_{ij,lm}(\hat{\mathbf{k}}) \equiv P_{il}(\hat{\mathbf{k}})P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{lm}(\hat{\mathbf{k}}) \ ,$$
$$P_{ij}(\hat{\mathbf{k}}) \equiv \delta_{ij} - \hat{\mathbf{k}}_i\hat{\mathbf{k}}_j \ , \qquad \hat{\mathbf{k}} \equiv \mathbf{k}/k \ ,$$

filters out the `TT` part of the Fourier transformed effective anisotropic stress tensor

$$\Pi_{ij}(\mathbf{k},\eta) = \Lambda_{ij,lm}(\hat{\mathbf{k}}) \int \frac{d^3q}{(2\pi)^3}\, q_l q_m\, \phi^a(\mathbf{q},\eta)\phi^a(\mathbf{k}-\mathbf{q},\eta) \ . \tag{3.5}$$

Note that we are summing over repeated indices both in coordinates and in field components.

The 2-point correlation function of the tensorial part of the anisotropic stress-tensor is of the form

$$\left\langle \Pi_{ij}(\mathbf{k},\eta)\Pi^*_{lm}(\mathbf{k}',\eta')\right\rangle \equiv (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\Pi^2(k,\eta,\eta')\mathcal{M}_{ijlm}(\hat{\mathbf{k}}) \ , \tag{3.6}$$

where

$$\mathcal{M}_{ijlm}(\hat{\mathbf{k}}) = \frac{1}{4}\left[\Lambda_{ij,lm}(\hat{\mathbf{k}}) + \Lambda_{ij,ml}(\hat{\mathbf{k}})\right] \ . \tag{3.7}$$

Since the trace $\mathcal{M}_{ijij} = 1$,

$$\left\langle \Pi_{ij}(\mathbf{k},\eta)\Pi^*_{ij}(\mathbf{k}',\eta')\right\rangle \equiv (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\Pi^2(k,\eta,\eta') \ . \tag{3.8}$$

To determine $\Pi^2(k,\eta,\eta')$, we compute $\left\langle \Pi_{ij}(\mathbf{k},\eta)\Pi^*_{ij}(\mathbf{k}',\eta')\right\rangle$ explicitly using Wick's theorem to reduce 4-point functions of the field to products of 2-point functions

$$\begin{aligned}
&\left\langle \Pi_{ij}(\mathbf{k},\eta)\Pi^*_{lm}(\mathbf{k}',\eta')\right\rangle = \\
&= \Lambda_{ij,pq}(\hat{\mathbf{k}})\,\Lambda_{lm,rs}(\hat{\mathbf{k}}')\int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\, q_p q_q q'_r q'_s \left\langle \phi^a(\mathbf{q},\eta)\phi^a(\mathbf{k}-\mathbf{q},\eta)\phi^{*b}(\mathbf{q}',\eta')\phi^{*b}(\mathbf{k}-\mathbf{q},\eta')\right\rangle \\
&= \int \frac{d^3q\, d^3q'}{(2\pi)^6}\left(q^{\mathrm{T}}\Lambda q\right)_{ij}\left(q'^{\mathrm{T}}\Lambda q'\right)_{lm}\Big[\left\langle \phi^a(\mathbf{q},\eta)\phi^{*a}(\mathbf{q}-\mathbf{k},\eta)\right\rangle\left\langle \phi^b(-\mathbf{q}',\eta')\phi^{*b}(\mathbf{k}'-\mathbf{q}',\eta')\right\rangle + \\
&\quad + \left\langle \phi^a(\mathbf{q},\eta)\phi^{*b}(\mathbf{q}',\eta')\right\rangle\left\langle \phi^a(\mathbf{k}-\mathbf{q},\eta)\phi^{*b}(\mathbf{k}'-\mathbf{q}',\eta')\right\rangle + \\
&\quad + \left\langle \phi^a(\mathbf{q},\eta)\phi^{*b}(\mathbf{k}'-\mathbf{q}',\eta')\right\rangle\left\langle \phi^a(\mathbf{k}-\mathbf{q},\eta)\phi^{*b}(\mathbf{q}',\eta')\right\rangle\Big] \\
&= \int d^3q\, d^3q'\left(q^{\mathrm{T}}\Lambda q\right)_{ij}\left(q'^{\mathrm{T}}\Lambda q'\right)_{lm}\Big[\mathcal{P}^{aa}_\phi(|\mathbf{q}|,\eta,\eta)\mathcal{P}^{bb}_\phi(|\mathbf{q}'|,\eta',\eta')\,\delta(\mathbf{k})\delta(\mathbf{k}') \\
&\quad + \mathcal{P}^{ab}_\phi(|\mathbf{q}|,\eta,\eta')\mathcal{P}^{ab}_\phi(|\mathbf{k}-\mathbf{q}|,\eta,\eta')\,\delta(\mathbf{q}-\mathbf{q}')\delta(\mathbf{k}-\mathbf{q}-\mathbf{k}'+\mathbf{q}') \\
&\quad + \mathcal{P}^{ab}_\phi(|\mathbf{q}|,\eta,\eta')\mathcal{P}^{ab}_\phi(|\mathbf{k}-\mathbf{q}|,\eta,\eta')\,\delta(\mathbf{q}'+\mathbf{q}-\mathbf{k}')\delta(\mathbf{q}'+\mathbf{q}-\mathbf{k})\Big]
\end{aligned} \tag{3.9}$$

where we use the notation $\left(q^{\mathrm{T}}\Lambda q\right)_{ij} \equiv q_l\Lambda_{ij,lm}q_m$ and we have introduced the reality condition $\phi^*(\mathbf{k}) = \phi(-\mathbf{k})$ and the unequal time correlator of the field ϕ which is defined in the same way as the one for β,

$$\langle\phi^a(\mathbf{k},\eta)\phi^{*b}(\mathbf{k}',\eta')\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')\,\mathcal{P}^{ab}_\phi(k,\eta,\eta') \ . \tag{3.10}$$

The zero-mode of the anisotropic stresses vanishes due to isotropy so that the first term in the square bracket of the integral (3.9) does not contribute.

We now can compute the unequal time correlator $\left\langle \Pi_{ij}(\mathbf{k},\eta)\Pi^*_{ij}(\mathbf{k}',\eta')\right\rangle$. Using

$$\left(q^{\mathrm{T}}\Lambda q\right)_{ij}\left(q^{\mathrm{T}}\Lambda q\right)_{ij} = \frac{1}{2}q^4\left(1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2\right)^2 \ , \tag{3.11}$$

we obtain

$$\Pi^2(k,\eta,\eta') = \int \frac{d^3q}{(2\pi)^3}\, q^4\left[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2\right]^2\mathcal{P}^{ab}_\phi(|\mathbf{q}|,\eta,\eta')\mathcal{P}^{ab}_\phi(|\mathbf{k}-\mathbf{q}|,\eta,\eta') \ . \tag{3.12}$$

We now relate the GW energy density spectrum to the unequal time anisotropic stress spectrum of the source, $\Pi^2(k,\eta,\eta')$. For this we first write the GW evolution equation in momentum space,

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} + k^2 h_{ij} = 8\pi G\,\Pi_{ij}\ . \tag{3.13}$$

Defining a new variable $\bar{h}_{ij} \equiv a h_{ij}$, one obtains

$$\bar{h}''_{ij} + \left(k^2 - \frac{a''}{a}\right)\bar{h}_{ij} = 8\pi G a\,\Pi_{ij}\ . \tag{3.14}$$

In a radiation dominated background ($a \propto \eta$) this reduces to

$$\bar{h}''_{ij} + k^2\bar{h}_{ij} = 8\pi G a\,\Pi_{ij}\ . \tag{3.15}$$

The solution of this differential equation with the initial conditions $h_{ij} = h'_{ij} = 0$ is given by the convolution of the source with the Green function $\mathcal{G}(k,\eta,\eta') = \sin(k\eta - k\eta')$,

$$\bar{h}_{ij}(\mathbf{k},\eta < \eta_{\rm fin}) = \frac{8\pi G}{k^2}\int_{x_*}^{x} dy\ a(y/k)\,\Pi_{ij}(\mathbf{k},y/k)\sin(x-y)\,, \tag{3.16}$$

where we have set $x \equiv k\eta$ and $y \equiv k\eta'$. The source of gravity waves is acting for a time interval $\delta\eta_* = (\eta_{\rm fin} - \eta_*) = \epsilon\eta_*$. If $\epsilon < 1$ we call the process short-lasting. This is the relevant case for example for GWs produced during a symmetry breaking phase transition where the source disappears after the phase transition since the latter typically lasts only for a fraction of the Hubble time. However, the Goldstone modes considered in this work may very well be long lived as they are not expected to interact with ordinary matter. In this case therefore a long lasting source may be better motivated. We discuss both cases below.

After the source has decayed, GWs are freely propagating, and thus described by the homogeneous solution of Eq. (3.15),

$$\bar{h}_{ij}(\mathbf{k},\eta > \eta_{\rm fin}) = A_{ij}(\mathbf{k})\sin(k\eta - k\eta_{\rm fin}) + B_{ij}(\mathbf{k})\cos(k\eta - k\eta_{\rm fin})\,. \tag{3.17}$$

The coefficients A_{ij} and B_{ij} are fixed by matching the homogeneous solution to the inhomogeneous one at $\eta = \eta_{\rm fin}$. Matching both $\bar{h}_{ij}$ and its derivative $\bar{h}'_{ij}$ yields

$$\begin{aligned} A_{ij}(\mathbf{k}) &= \frac{8\pi G}{k^2}\int_{x_*}^{x_{\rm fin}} dy\ a(y/k)\Pi_{ij}(\mathbf{k},y/k)\cos(x_{\rm fin}-y)\ ,\\ B_{ij}(\mathbf{k}) &= \frac{8\pi G}{k^2}\int_{x_*}^{x_{\rm fin}} dy\ a(y/k)\Pi_{ij}(\mathbf{k},y/k)\sin(x_{\rm fin}-y)\ . \end{aligned} \tag{3.18}$$

The GW energy density is given by (see *e.g.* [6])

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^3|h'|^2(k,\eta)}{2(2\pi)^3 G a^2}\ , \tag{3.19}$$

where the GW power spectrum has been normalized as follows:

$$\Big\langle h'_{ij}(\mathbf{k},\eta)h'^*_{ij}(\mathbf{q},\eta)\Big\rangle = 2\Big\langle h'_+(\mathbf{k},\eta)h'^*_+(\mathbf{q},\eta) + h'_\times(\mathbf{k},\eta)h'^*_\times(\mathbf{q},\eta)\Big\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{q})\,|h'|^2(k,\eta)\ . \tag{3.20}$$

Here our normalization differs from that of Ref. [27]. Their definition of the power spectrum is related to ours by

$$\mathcal{P}(k,\eta) \equiv 2\pi k^3 |h|^2(k,\eta) \tag{3.21}$$

and they infer $\frac{d\Omega_{GW}(k,\eta_0)}{d\log k} = \frac{k^2\mathcal{P}(k,\eta)}{6H_0^2}$ whereas we obtain, with (3.19) and $h' = kh$ for sub-horizon modes,

$$\frac{d\Omega_{GW}(k,\eta_0)}{d\log k} = \frac{k^5|h|^2(k,\eta)}{6\pi^2 H_0^2} = \frac{k^2\mathcal{P}(k,\eta)}{12\pi^3 H_0^2} \ .$$

This difference in the normalization, which we attribute to an error in Ref. [27], leads to a reduction of the final result by about a factor 60, which may be relevant for observations.

With the solution for $\bar{h}_{ij}$ above, we obtain for $\eta > \eta_{\rm fin}$

$$\begin{aligned} |h'|^2(k,\eta) &= \frac{1}{2a^2}\left(k^2+\mathcal{H}^2\right)\left(\langle A_{ij}A^*_{ij}\rangle + \langle B_{ij}B^*_{ij}\rangle\right) \\ &= \frac{k^2+\mathcal{H}^2}{2a^2}\left(\frac{8\pi G}{k^2}\right)^2 \int_{x_*}^{x_{\rm fin}} dy \int_{x_*}^{x_{\rm fin}} dz\ a\left(\frac{y}{k}\right) a\left(\frac{z}{k}\right)\cos(z-y)\Pi^2\left(k,\frac{y}{k},\frac{z}{k}\right), \end{aligned} \tag{3.22}$$

where we have used Eq. (3.8). The GW energy density at time η is of course well defined only for waves with a wavelength well within the horizon, $k \gg \mathcal{H}$. Therefore we shall approximate $k^2+\mathcal{H}^2 \simeq k^2$ in the following.

The GWs are sourced by the anisotropic stress of the scalar field $\phi^a = v\beta^a$. The correlators are simply related by

$$\mathcal{P}_\phi^{ab} = v^2 \mathcal{P}_\beta^{ab} \ .$$

With Eq. (3.12) we obtain the following expression for the GW energy density after the decay of the source, $\eta > \eta_{\rm fin}$,

$$\begin{aligned} \frac{d\rho_{\rm GW}(k,\eta)}{d\log k} &= \frac{G\,v^4}{4\pi^4}\frac{k^3}{a^4(\eta)}\int_{\eta_*}^{\eta_{\rm fin}} d\tau \int_{\eta_*}^{\eta_{\rm fin}} d\xi\ a(\tau)a(\xi)\ \cos(k\xi - k\tau) \\ &\quad \times \int d^3p\ p^4 \sin^4\theta\ \mathcal{P}_\beta^{ab}(p,\tau,\xi)\,\mathcal{P}_\beta^{ab}(|\mathbf{k}-\mathbf{p}|,\tau,\xi)\,, \end{aligned} \tag{3.23}$$

where $\cos\theta \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$. Inserting the power spectrum of β in the above expression and summing over the field components, we find

$$\begin{aligned} \frac{d\rho_{\rm GW}(k,\eta)}{d\log k} &= \frac{G\,v^4}{4\pi^4}\frac{k^3}{a^4(\eta)}\frac{36\pi^4A^2}{N}\int_{\eta_*}^{\eta_{\rm fin}} d\tau \int_{\eta_*}^{\eta_{\rm fin}} d\xi\ a(\tau)a(\xi)\ \cos(k\xi - k\tau) \\ &\quad \times \int_{\substack{p\,<\,1/\eta_* \\ |\mathbf{k}-\mathbf{p}|\,<\,1/\eta_*}} d^3p\ p^4\sin^4\theta\ \tau^3\xi^3\ \frac{J_\nu(p\tau)}{(p\tau)^\nu}\frac{J_\nu(p\xi)}{(p\xi)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\tau)}{(|\mathbf{k}-\mathbf{p}|\tau)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\xi)}{(|\mathbf{k}-\mathbf{p}|\xi)^\nu}\,. \end{aligned} \tag{3.24}$$

Here the constant A comes from the normalization of β, and it is given by Eq. (2.14). In the radiation dominated background considered here, we have $\nu = 1+\gamma = 2$ and $A = 5\pi/4$. Note also that we choose the normalization of the scale factor such that $a(\eta_0) = 1$. Hence the *comoving* wave number k is simply related to the present frequency of the GW by

$$f = \frac{k}{2\pi} \ .$$

In the next section we evaluate the present amplitude and frequency dependence of the GW spectrum generated in this way explicitly. For this, the following relation between temperature and time in a radiation dominated Universe are useful [34],

$$H^2(t) = \frac{1}{\eta^2\, a(\eta)^2} = \frac{8\pi G}{3}\frac{\pi^2}{30}\, g_{\rm eff}(\eta) T^4(\eta)\,. \tag{3.25}$$

Assuming an adiabatic expansion, $g_{\rm eff}(aT)^3 = $ const., one finds

$$\eta = \frac{M_{\rm Pl}}{T(\eta)T_0}\left(\frac{g_{\rm eff}(\eta)}{2}\right)^{1/3}\left(\frac{45}{4\pi^3 g_{\rm eff}(\eta)}\right)^{1/2} = 1.6\times 10^7 {\rm sec}\left(\frac{\rm GeV}{T}\right) g_{\rm eff}^{-1/6}(T)\ . \tag{3.26}$$

On the other hand, the expression for the temperature associated to a global $\mathcal{O}(N)$ symmetry breaking is [33]

$$T_* = \sqrt{\frac{24}{N+2}}v\ , \tag{3.27}$$

independent of the coupling λ.

Before moving to the evaluation of Eq. (3.24), let us briefly determine the frequencies for the GW sources discussed in this paper. We are studying the IR modes $k\eta_* < 1$ of the GW spectrum, corresponding to frequencies smaller than the expansion rate at the time of production, $f_* = \mathcal{H}_*/(2\pi)$,

$$f_* = \frac{1}{2\pi\eta_*} \approx 10^{-8}\left(\frac{T_*}{\rm GeV}\right)\ {\rm Hz}\ . \tag{3.28}$$

For the EW scale this corresponds to $f_*^{\rm EW} \sim 10^{-6}$ Hz, while for the GUT scale the associated frequency is $f_*^{GUT} \sim 10^8$ Hz. For a given energy scale $M \simeq T_*$ at the time of production, we are describing one frequency range or another, but always frequencies smaller than the one corresponding today to that energy scale, $f < f_*(M) \sim 10^{-8}{\rm Hz}(M/{\rm GeV})$. Clearly, only processes taking place in the radiation dominated Universe generate GWs with sufficiently high frequencies such that they can be observed by direct GW detection experiments. Indeed the frequency associated to the horizon at the matter-radiation equality is far too small, $f_*^{\rm eq} \sim 10^{-17}$ Hz, to be observed by direct GW detectors, like LIGO, LISA or BBO will be working. Therefore we consider only processes in the radiation dominated Universe and $\gamma = 1$ and $\nu = 2$ are assumed for the rest of the paper.

4. The gravitational wave spectrum today

In this section we study two different cases, first the situation in which the source producing GWs lasts only a small fraction of the Hubble time at the moment of production and, second, the case in which the GW source acts for a much longer time, until the moment at which a given mode enters the horizon.

4.1 Short lived source

We first estimate the amplitude of the GW spectrum for large wavelengths, $k < \mathcal{H}_*$, from a short lived source which lasts from η_* to $\eta_{\rm fin}$, such that $(\eta_{\rm fin} - \eta_*)/\eta_* \equiv \epsilon \ll 1$ (as *e.g.* for the radial mode of ϕ in hybrid preheating [12, 14]). Let us first note the following facts:

1) From Eq. (3.24) we see immediately that for small wavenumbers, $k\eta_{\rm fin} \ll 1$, the result scales like

$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \ .$$

2) Since the source is short lived, $\eta_* \approx \eta_{\rm fin}$, and we deal with superhorizon modes, $k\eta_* \ll 1$, we may set $\cos(k\eta - k\eta') \approx 1$ and the time integral can be replaced simply by a factor $\epsilon\eta_*$.

3) To estimate the momentum integral, we use that Bessel functions at small arguments, $x \equiv k\eta < 1$, can be approximated by $J_\nu(x) \approx (x/2)^\nu/\Gamma(1+\nu)$. To obtain the dominant contribution at large wavelength (*i.e.* the least blue part) we may also set $|\mathbf{k} - \mathbf{q}|\eta_* \simeq q\eta_*$.

Using all the above considerations, we are left with a simple integral for the evaluation of the spectra of the IR modes ($k\eta_* \ll 1$) of GWs, at any time $\eta \gg \eta_*$ for which those modes have already crossed the horizon

$$\begin{aligned} \left.\frac{d\rho_{\rm GW}(\eta)}{d\log k}\right|_{k\eta_*\ll 1} &\simeq \frac{Gv^4}{4\pi^4} 36\pi^4 A^2 \frac{k^3}{a^4(\eta)}\frac{2\pi}{N}\int_{-1}^{1} d\cos\theta\,\sin^4\theta \int_0^{1/\eta_*} dp\, \frac{p^6}{2^{2\nu}\Gamma^4(\nu+1)} \\ &\times \left(\int_{\eta_*}^{\eta_{\rm fin}} d\tau\, a(\tau)\tau^3\right)^2 = \frac{3\cdot 5\pi^3}{7\cdot 2^{11}}\frac{Gv^4}{N}\left(\frac{a_*}{a(\eta)}\right)^4 \epsilon^2 H_*^2\,(k\eta_*)^3\,, \end{aligned} \tag{4.1}$$

where we used $A = 5\pi/4$, $\nu = 2$ and we approximated $\int_{\eta_*}^{\eta_{\rm fin}} d\tau a(\tau)\tau^3 \approx a(\eta_*)\eta_*^3\delta\eta_* = \epsilon\, a(\eta_*)\eta_*^4$, since we have set $\eta_{\rm fin} - \eta_* = \delta\eta_* \simeq \epsilon\eta_*$.

With this we can now evaluate the ratio of the GW energy density to the critical density today, for the IR modes $k\eta_* \ll 1$, as

$$\begin{aligned} \Omega_{\rm GW}(f) &= \frac{1}{\rho_c}\frac{d\rho_{\rm GW}(\eta_0)}{d\log k} \approx \frac{5\pi^4}{7\cdot 2^8}\left(\frac{v}{M_{\rm Pl}}\right)^4 \frac{\epsilon^2}{N}\,\Omega_{\rm rad}(k\eta_*)^3 \\ &\sim 10^{-5}\left(\frac{v}{M_{\rm Pl}}\right)^4 \frac{\epsilon^2}{N}\,(k\eta_*)^3\ , \end{aligned} \tag{4.2}$$

where we used $H_*^2 = 8\pi G\rho_*/3$, we expressed the radiation density today as $\rho_{\rm rad} \approx \rho_*(a_*/a_0)^4$ and we introduced the the radiation density parameter today as $\Omega_{\rm rad} \approx 4.2 \times 10^{-5}$. We have also neglected the factors coming from the ratio of the effective relativistic degrees of freedom since they appear only with the power 1/3.

Note that this formula is general for the IR spectrum of GWs generated at any process in which the source, a N-component scalar field, has rapidly acquired its true *vev* v at η_* and undergoes a short phase of self-ordering which lasts for a fraction $\epsilon < 1$ of the Hubble time.

Finally, note also that very generically we have $\eta_* \propto T_*^{-1} \propto 1/v$ so that $\Omega_{\rm GW} \propto v^4\eta_*^3 k^3 \propto v\, k^3$ and not as v^4, as one could naively have concluded from Eq. (4.2).

4.1.1 The electroweak phase transition

The comoving horizon size at the electroweak (EW) phase transition is given by the EW energy scale $T_* \sim 100$ GeV, $g_{\rm eff}(T_*) = 106.75$,

$$\eta_* \simeq 7.5 \times 10^4 \text{ sec}\ .$$

Inserting this above with $f = k/(2\pi)$, we find

$$\Omega_{\rm GW}(f) \approx 4.2 \times 10^5 \, \frac{5\pi^4(2\pi)^3}{7 \cdot 2^8} \, \Omega_{\rm rad} \left(\frac{v}{M_{\rm Pl}}\right)^4 \frac{\epsilon^2}{N} \left(\frac{f}{\rm mHz}\right)^3 \sim 10^{-65} \, \frac{\epsilon^2}{N} \left(\frac{f}{\rm mHz}\right)^3 \ . \quad (4.3)$$

For the last expression we have used $v \simeq T_*$. This result is of course unmeasurably small.

4.1.2 A GUT scale phase transition

To have any chance to measure this spectrum, we need a *vev* which is not too many orders of magnitude below that Planck scale, since the GW energy density is suppressed by a fourth power of the ratio of the *vev* to $M_{\rm Pl}$. The best change might be a GUT scale with a *vev* of the order of $v \simeq 10^{16}$GeV. But then of course η_* will be very small and the dominant contribution will come from very high frequencies, lower frequencies being suppressed by the factor $(k\eta_*)^3$. For $T_* = 10^{16}$GeV we have

$$\eta_* \simeq 5 \times 10^{-10} \ {\rm sec} \ ,$$

leading to

$$\Omega_{\rm GW}(f) \approx 0.125 \, \frac{5\pi^4(2\pi)^3}{7 \cdot 2^8} \, \Omega_{\rm rad} \left(\frac{v}{M_{\rm Pl}}\right)^4 \frac{\epsilon^2}{N} \left(\frac{f}{\rm GHz}\right)^3 \sim 10^{-16} \, \frac{\epsilon^2}{N} \left(\frac{f}{\rm GHz}\right)^3 \ . \quad (4.4)$$

Apart from the fact that this result suffers severe additional suppression at measurable frequencies which are significantly below 1GHz $= 10^9$Hz, the sensitivity of $10^{-12}\Omega_{\rm rad} \simeq 10^{-16}$ cannot be reached with any presently proposed experiment at those frequencies.

Therefore, we can only conclude that the superhorizon GW spectrum generated from a short lived self ordering scalar field is much below presently proposed experimental sensitivities.

4.2 A long lived source

As we have seen in the previous subsection, short lived Goldstone modes cannot lead to a significant GW background. But since Goldstone modes are typically non-interacting and long lived, it is more natural to consider them for a time which is much longer than the horizon scale η_*. To compute the GW energy density produced by such a self ordering scalar field, we consider Eq. (3.24) and set $\eta_{\rm fin} = \eta_k \equiv 1/k$, since the solution (2.15) decays inside the horizon, when $k\eta > 1$. We then have to compute the following integral

$$\frac{d\rho_{\rm GW}(k,\eta_k)}{d\log k} = \frac{G\, v^4}{4\pi^4} \frac{k^3}{a^4(\eta_k)} \frac{36\pi^4 A^2}{N} \int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{1/k} d\xi \ a(\tau)a(\xi) \cos(k\xi - k\tau) \times$$
$$\int_{\substack{p\eta_* < 1 \\ |\mathbf{p}-\mathbf{k}|\eta_* < 1}} d^3p \ p^4 \sin^4\theta \ \tau^3\xi^3 \, \frac{J_\nu(p\tau)}{(p\tau)^\nu} \frac{J_\nu(p\xi)}{(p\xi)^\nu} \frac{J_\nu(|\mathbf{k}-\mathbf{p}|\tau)}{(|\mathbf{k}-\mathbf{p}|\tau)^\nu} \frac{J_\nu(|\mathbf{k}-\mathbf{p}|\xi)}{(|\mathbf{k}-\mathbf{p}|\xi)^\nu} \ , \quad (4.5)$$

Note that the range of integration of the variable p in the above expression is set to be $\{p\eta_* < 1, \ |\mathbf{p}-\mathbf{k}|\eta_* < 1\}$ since the initial two point correlator of the scalar field turns out to be different from zero only in this range of momenta [*c.f.* Eq. (2.9)].

In order to obtain an analytical result for the above integral, we perform the following approximations:

- We are interested in scales k that are superhorizon for all the time of GW production, namely $k\tau < 1$ and $k\xi < 1$ for times τ, ξ between η_* and $\eta_{\rm fin} = 1/k$, therefore we approximate $\cos(k\xi - k\tau) \simeq 1$.

- We neglect the angular dependence of $|\mathbf{p} - \mathbf{k}|$ so that the angular integral reduces to $2\pi \int \sin^4\theta d\cos\theta = 32\pi/15$.

- In the range of integration where $p\tau \gg 1$ we substitute $|\mathbf{k} - \mathbf{p}|\tau \simeq p\tau$, while when $p\tau \ll 1$ we approximate $|\mathbf{k} - \mathbf{p}|\tau \ll 1$.

- The range of momenta for which we can expand the Bessel functions in terms of small arguments is $p < \min(1/\tau, 1/\xi)$, while in the range $\min(1/\tau, 1/\xi) < p < \max(1/\tau, 1/\xi)$ we should distinguish between large and small argument expansions of the Bessel functions. Finally, in the range $\max(1/\tau, 1/\xi) < p < 1/\eta_*$ one can consider the large argument limit for all the four Bessel functions of the above integral.

Taking into account all the above considerations, we find that the complete integral becomes

$$\int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{1/k} d\xi \int_0^\infty dp\, f(p,\tau,\xi) \;=\; 2\int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi \left(\int_0^{1/\tau} dp\, f + \int_{1/\tau}^{1/\xi} dp\, f + \int_{1/\xi}^{1/\eta_*} dp\, f \right) ,$$

which allows us to separate the integral in p using the asymptotic behaviour of the Bessel functions,

$$J_\nu(x) \simeq \frac{x^\nu}{2^\nu \Gamma(\nu+1)} \qquad \text{for } x \ll 1 ,$$
$$J_\nu(x) \simeq \sqrt{\frac{2}{x\pi}} \cos\left(x - \frac{(2\nu+1)\pi}{4} \right) \qquad \text{for } x \gg 1 .$$

We can distinguish three different intervals:

- The IR contribution, $I_1(k)$, for $0 < p < 1/\tau$, with $|\mathbf{k} - \mathbf{p}|\tau < 1$ and $|\mathbf{k} - \mathbf{p}|\xi < 1$.

- The mixed (UV+IR) contribution, $I_2(k)$, for $1/\tau < p < 1/\xi$, with $|\mathbf{k} - \mathbf{p}|\tau \simeq p\tau > 1$ but $|\mathbf{k} - \mathbf{p}|\xi \simeq p\xi < 1$.

- The UV contribution, $I_3(k)$, for $1/\xi < p < 1/\eta_*$, with $|\mathbf{k} - \mathbf{p}|\tau \simeq p\tau > 1$ and $|\mathbf{k} - \mathbf{p}|\xi \simeq p\xi > 1$.

Therefore we can finally write

$$\frac{d\rho_{\rm GW}(k,\eta_k)}{d\log k} = \mathcal{D}(k)\left[I_1(k) + I_2(k) + I_3(k)\right] , \tag{4.6}$$

where the pre-factor $\mathcal{D}(k)$ contains the coefficients in front of the integral in Eq. (4.5), the factor coming from the angular integration $(32\pi/15)$ and the factor 2 that comes from the symmetry of the double time integration, namely

$$\mathcal{D}(k) \equiv \frac{G\,v^4}{4\pi^4} \frac{k^3}{a^4(\eta_k)} \frac{36\pi^4 A^2}{N} \times \frac{32\pi}{15} \times 2 = \frac{G\,v^4}{N} \frac{k^3}{a^4(\eta_k)} 15 \cdot 4\pi^3 . \tag{4.7}$$

The three integrals of Eq. (4.6) are given by

$$\begin{aligned}
I_1(k) &\equiv \int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi \; a(\tau)\; a(\xi)\; \tau^3\; \xi^3 \int_0^{1/\tau} dp\; p^6 \; \frac{J_\nu(p\tau)}{(p\tau)^\nu}\frac{J_\nu(p\xi)}{(p\xi)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\tau)}{(|\mathbf{k}-\mathbf{p}|\tau)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\xi)}{(|\mathbf{k}-\mathbf{p}|\xi)^\nu} \\
&\simeq \frac{H_0^2\Omega_{\rm rad}}{4096}\int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi\; \tau^4\; \xi^4 \int_0^{1/\tau} dp\; p^6 \\
&= \frac{H_0^2\Omega_{\rm rad}}{4096\; k^3}\frac{1}{35}\left[\frac{1}{3}-\frac{5}{6}(k\eta_*)^3+\frac{1}{2}(k\eta_*)^5\right] , \qquad (4.8)
\end{aligned}$$

$$\begin{aligned}
I_2(k) &\equiv \int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi \; a(\tau)\; a(\xi)\; \tau^3\; \xi^3 \int_{1/\tau}^{1/\xi} dp\; p^6 \; \frac{J_\nu(p\tau)}{(p\tau)^\nu}\frac{J_\nu(p\xi)}{(p\xi)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\tau)}{(|\mathbf{k}-\mathbf{p}|\tau)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\xi)}{(|\mathbf{k}-\mathbf{p}|\xi)^\nu} \\
&\simeq \frac{H_0^2\Omega_{\rm rad}}{32\pi}\int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi\; \tau^4\; \xi^4 \int_{1/\tau}^{1/\xi} \frac{dp\; p^6}{(p\tau)^5}\; \cos^2\left(p\tau-\frac{5\pi}{4}\right) \\
&= \frac{H_0^2\Omega_{\rm rad}}{128\pi\; k^3}\left[\frac{2}{45}+\frac{1}{18}(k\eta_*)^3-\frac{1}{10}(k\eta_*)^5+\frac{(k\eta_*)^3}{3}\log(k\eta_*)\right] , \qquad (4.9)
\end{aligned}$$

and

$$\begin{aligned}
I_3(k) &\equiv \int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi \; a(\tau)\; a(\xi)\; \tau^3\; \xi^3 \int_{1/\xi}^{1/\eta_*} dp\; p^6 \; \frac{J_\nu(p\tau)}{(p\tau)^\nu}\frac{J_\nu(p\xi)}{(p\xi)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\tau)}{(|\mathbf{k}-\mathbf{p}|\tau)^\nu}\frac{J_\nu(|\mathbf{k}-\mathbf{p}|\xi)}{(|\mathbf{k}-\mathbf{p}|\xi)^\nu} \\
&\simeq \frac{4\,H_0^2\Omega_{\rm rad}}{\pi^2}\int_{\eta_*}^{1/k} d\tau \int_{\eta_*}^{\tau} d\xi\; \tau^4\; \xi^4 \int_{1/\xi}^{1/\eta_*} \frac{dp\; p^6}{(p\tau)^5(p\xi)^5}\cos^2\left(p\tau-\frac{5\pi}{4}\right)\cos^2\left(p\xi-\frac{5\pi}{4}\right) \\
&= \frac{H_0^2\Omega_{\rm rad}}{3\pi^2\; k^3}\left[\frac{1}{9}-\frac{1}{9}(k\eta_*)^3-(k\eta_*)^3\left(\frac{1}{2}\log^2(k\eta_*)-\frac{1}{3}\log(k\eta_*)\right)\right] . \qquad (4.10)
\end{aligned}$$

More precisely, in the above computation we substituted each $\cos^2 x$ by its mean value $\langle\cos^2 x\rangle = 1/2$ averaged over a few oscillations, and we introduced the usual expression for the scale factor in a radiation dominated background, $a(\eta) \simeq H_0\sqrt{\Omega_{\rm rad}}\eta$, which is consistent with $a_0 = 1$ today.

All three terms have a scale-invariant spectrum. Actually, the "UV" contribution given in Eq. (4.10) is the largest. Summing all the three contribution and considering the dominant part in the limit $k\eta_* \ll 1$ [hence also $(k\eta_*)^3\log(k\eta_*) \ll 1$], we obtain the following scale-invariant spectrum

$$\begin{aligned}
\frac{d\rho_{\rm GW}(k,\eta_k)}{d\log k} &\simeq 5\cdot 2^5\pi^4\frac{\Omega_{\rm rad}\,\rho_c}{Na^4(\eta_k)}\left(\frac{v}{M_{\rm Pl}}\right)^4\left(\frac{1}{2^{12}\cdot 105}+\frac{1}{2^6\pi\cdot 45}+\frac{1}{27\pi^2}\right) \\
&\simeq 60\times\frac{\Omega_{\rm rad}\,\rho_c}{Na^4(\eta_k)}\left(\frac{v}{M_{\rm Pl}}\right)^4 , \qquad (4.11)
\end{aligned}$$

where we have used the Friedmann equation $H_0^2 = 8\pi G\rho_c/3$. Redshifting the above expression until today, we obtain for the GW energy density parameter,

$$\Omega_{\rm GW}(k,\eta_0) \equiv \frac{d\rho_{\rm GW}(k,\eta_0)}{\rho_c d\log k} = \frac{d\rho_{\rm GW}(k,\eta_k)}{\rho_c d\log k}a^4(\eta_k) \simeq \frac{60}{N}\;\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4 . \qquad (4.12)$$

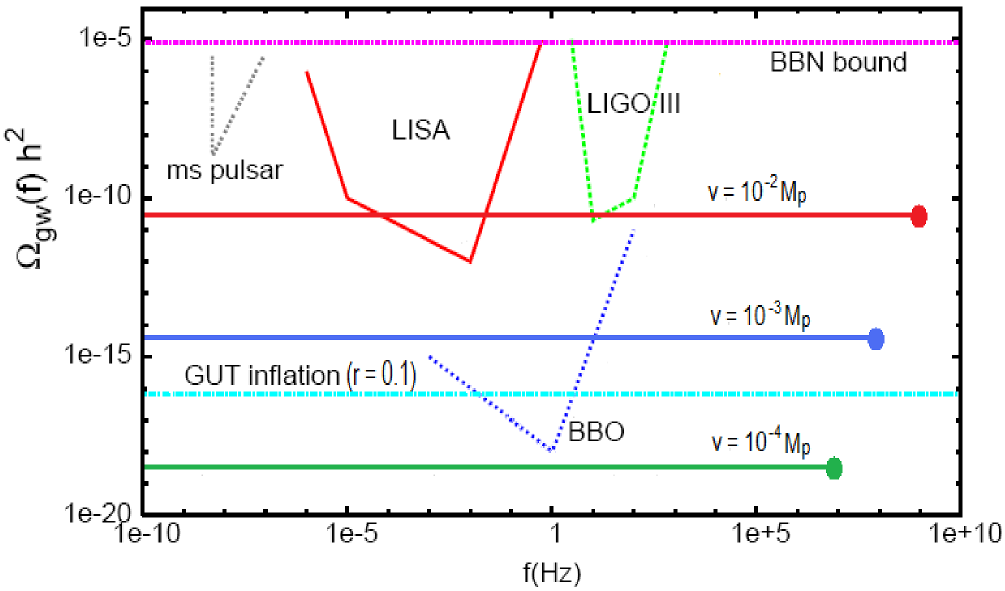

Figure 1: The sensitivity of present and future GW experiments are compared with our results for a long lasting source and inflation. We show, the amplitude of the scale-invariant GW background expected from a GUT scale inflation (blue, dashed) and from a self-ordering long lived source as studied in this paper, for a symmetry breaking field with $N = 4$ real components and a *vev* $v = 10^{-2}M_{\rm Pl}$ (top, red line), $v = 10^{-3}M_{\rm Pl}$ (middle, blue line, overlying with inflation) and $v = 10^{-4}M_{\rm Pl}$ (bottom, green line). The big dot at the right end of the horizontal lines represents the frequency (3.28) associated to the horizon at the initial time of production.

This corresponds to a scale-invariant GW spectrum produced by a self-ordering scalar field in the large N-limit. This result is valid for all wave numbers k which enter the horizon when the Goldstone modes of our N-component field are still massless and the field has not yet decayed. Scales which enter the horizon after this time $\eta_{\rm fin}$, *i.e.* scales with $k\eta_{\rm fin} < 1$, are suppressed by a factor $(k\eta_{\rm fin})^3$, as for them the result for a short lived source with η_* replaced by $\eta_{\rm fin}$ applies.

4.3 Numerical integration

In order to obtain more accurate results, and to check the validity of our analytical approximations, we have also performed a numerical evaluation of the integrals in Eq. (3.24). If we set the final time of integration to be the horizon crossing, $\eta_{\rm fin} = 1/k$, as we did in the analytical evaluation for the long lasting source (4.5), we obtain the following result for the final GW density parameter today

$$\Omega_{\rm GW}(k,\eta_0) \simeq \frac{22}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4 , \qquad \eta_{\rm fin} = 1/k\,. \tag{4.13}$$

This suggests that the analytical approximation somewhat overestimates the result. However, we can continue the integration to later times when the wavelength has already entered the horizon.

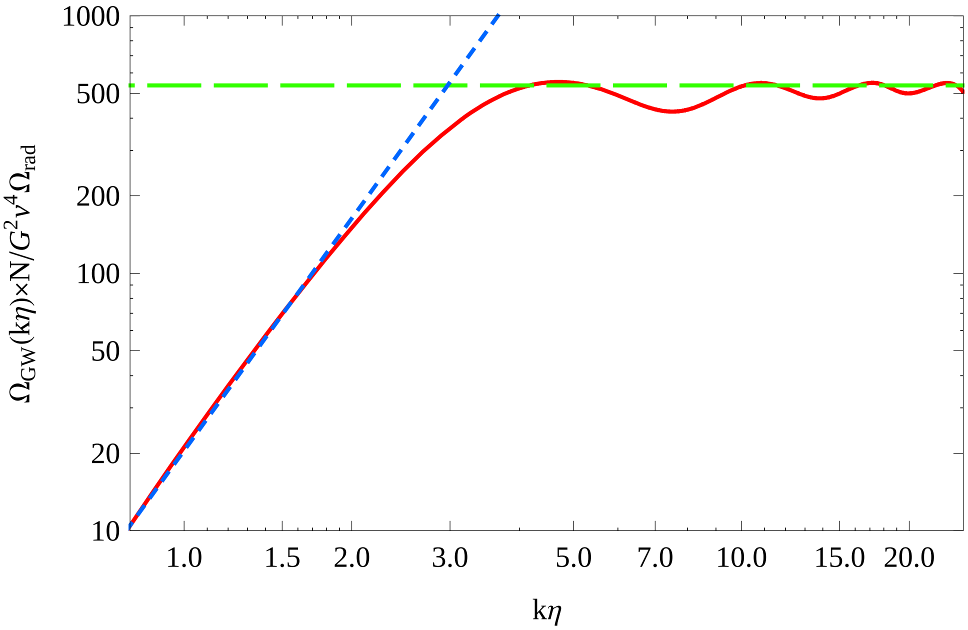

Figure 2: The density parameter in gravitational waves as a function of $k\eta$. For scales outside the horizon, $k\eta < \pi$, we observe the $(k\eta)^3$ dependence (short dashed line), while for scales that have entered the horizon, $k\eta > \pi$, the GW energy density saturates, at a normalized value of 511 (long dashed line). This result implies a significant scale-invariant GW spectrum today.

The integral in Eq. (4.5) allows us to compute the GW energy density in the limit $k\eta_* \ll 1$, using the change of variables $u = \cos\theta,\ q = p/k,\ x = k\tau$,

$$\Omega_{\rm GW}(k,\eta) = \frac{G^2 v^4 \Omega_{\rm rad}}{N a^4(\eta)}\, 75\pi^4 \int_0^\infty dq\, q^2 F(q) \left\{ \left[\int_0^{k\eta} dx\ \cos x\, J_2^2(qx)\right]^2 + \left[\int_0^{k\eta} dx\ \sin x\, J_2^2(qx)\right]^2 \right\} \tag{4.14}$$

where the kernel $F(q)$ comes from the integration over angles,

$$F(q) = \int_{-1}^{1} \frac{du\,(1-u^2)^2}{(q^2+1-2qu)^2} = \frac{1}{24q^5}\left[16q + 12q(q^2-1)^2 + 3(q^2-1)^2(q^2+1)\log\frac{(q-1)^2}{(q+1)^2}\right]$$

and we have made the approximation, $J_2(x\sqrt{q^2+1-2qu}) \to J_2(qx)$, inside the time integration. We have checked that for large times the result is correct within 0.1%.

Numerically evaluating (4.14), we find that the GW energy density continues to grow until horizon crossing, $k\eta \simeq \pi$, and saturates thereafter, see Fig. 2. This agrees with the result of Ref. [27], who find a peak in the power spectrum $\mathcal{P}(k,\eta)$ at approximately this value, and also explains the $1/a(\eta)^2$ dependence of the Power spectrum, $\mathcal{P} \propto \Omega_{\rm GW}/a^2$, for scales that have already entered the horizon.

For $k\eta \gg 4$ the gravitational wave energy density saturates at a value

$$\Omega_{\rm GW}(k,\eta_0) \simeq \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4, \tag{4.15}$$

where we used again the usual normalization of the scale factor in a radiation dominated background. These results suggest that the GW spectrum produced by this mechanism still grows inside the horizon and reaches its final value somewhat after horizon crossing. This is consistent with the fact that the power of the scalar field that sources these GWs is not absent inside the horizon, but it is indeed given by the Bessel functions in Eq. (2.17), which decay rather slowly as functions of $k\eta$.

In the following analysis we will consider the numbers arising from the numerical integration, as given in Eq. (4.15).

4.4 Observational constraints

Our result for the amplitude of the GW spectrum (4.15) is inside the range of detectability of the BBO [31] experiment ($\Omega_{\rm GW}(k) \gtrsim 10^{-17}$) and is marginally detectable by LISA [30] or advanced LIGO [29] ($\Omega_{\rm GW}(k) \gtrsim 10^{-10}$). Indeed, with $\Omega_{\rm rad} \simeq 4.2 \times 10^{-5}$, we find that BBO would detect this signal if the symmetry breaking scale v satisfies

$$\left(\frac{v}{M_{\rm Pl}}\right)^4 \gtrsim 4.7 \cdot 10^{-16} N \qquad \Rightarrow \qquad \frac{v}{M_{\rm Pl}} \gtrsim 1.5 \cdot 10^{-4} N^{1/4} \ .$$

Concerning the sensitivity of LIGO or LISA, the signal is detectable if

$$\left(\frac{v}{M_{\rm Pl}}\right)^4 \gtrsim 4.7 \cdot 10^{-9} N \qquad \Rightarrow \qquad \frac{v}{M_{\rm Pl}} \gtrsim 0.008\, N^{1/4} \ .$$

In other words, for scales higher or around the GUT scale, $v \gtrsim 10^{16}$GeV, the very long wavelength tail which we have studied here could be observed.

In order to relate the above scale-invariant GW energy density to the GW spectrum from inflation, we compute the relative tensor-to-scalar ratio r. Following Ref. [35], one has the following expression for the GW density parameter from inflation

$$\Omega_{\rm GW}(k,\eta_0) = 4.36 \times 10^{-15}\, r \left(\frac{k}{k_0}\right)^{n_{\rm T}} \ , \qquad r \equiv \frac{\mathcal{P}_{\rm T}(k_0)}{\mathcal{P}_{\rm S}(k_0)} \ , \tag{4.16}$$

where $k_0 = 0.002\, h\,{\rm Mpc}^{-1}$, $\mathcal{P}_{\rm T}(k) = r\mathcal{P}_{\rm S}(k_0)(k/k_0)^{n_{\rm T}}$ and we used the WMAP result, $\mathcal{P}_{\rm S}(k_0) = 2.21 \times 10^{-9}$. This concerns only the wavelengths which enter the horizon in the radiation dominated era, before equality. Comparing the above expression for $n_T \simeq 0$ with our Eq. (4.15), we obtain in our case

$$r \simeq \frac{3}{N}\left(\frac{v}{10^{16}{\rm GeV}}\right)^4 . \tag{4.17}$$

Another usefull comparison with inflation is the relative strength of the GW energy densities produced by the above two different mechanisms. Considering always wavelengths which enter the horizon in the radiation dominated epoch, we have [36]

$$\Omega_{GW}^{(\rm inf)} = 10^{-13}\left(\frac{H_*}{10^{-4}M_{\rm Pl}}\right)^2 = 8.4 \times 10^{-5}\left(\frac{M}{M_{\rm Pl}}\right)^4 , \tag{4.18}$$

where M denotes the energy scale of inflation, $H_*^2 \equiv 8\pi G M^4/3$. The ratio between the GW energy density produced by our mechanism and the one from inflation is then

$$\mathcal{R} \equiv \frac{\Omega_{\rm GW}(k,\eta_0)}{\Omega_{\rm GW}^{(\rm inf)}(k,\eta_0)} \simeq \frac{256}{N}\left(\frac{v}{M}\right)^4 . \tag{4.19}$$

Comparing these results with those of Ref. [27], where the authors mainly concentrate on the spectrum of GWs produced in a matter dominated universe, we reproduce perfectly the amplitude of their spectrum $\mathcal{P}(k,\eta)$ defined as in Eq. (3.21), but their final relative strength $\mathcal{R}$ is nearly 2 orders of magnitude larger than what we find in Eq. (4.19). We believe this is due to the factor $1/(2\pi^3)$ missing in their expression for $\Omega_{\rm GW}(k,\eta_0)$ which has to be introduced for consistency with the definition of the power spectrum $\mathcal{P}(k)$.

5. Conclusions

In this paper we have estimated the contributions to the gravitational wave background from a symmetry breaking phase transition on large scales, $k\eta_* < 1$. We have concentrated on the analysis of the Goldstone modes and we obtained the following main conclusions.

If the modes are *short lived* with duration $\epsilon\eta_*$, $\epsilon < 1$ their contribution is blue and suppressed by a factor $\epsilon^2(k\eta_*)^3$. This result is actually generic, independent of the nature of the short lived source. Indeed, one typically obtains

$$\Omega_{\rm GW}(k) \simeq (k\eta_*)^3\Omega_{\rm rad}\Omega_X^2\epsilon^2 \ , \tag{5.1}$$

where Ω_X is the density parameter of the source of anisotropic stresses at the moment of creation. For the Goldstone modes the factor Ω_X^2 is replaced by $(v/M_{\rm Pl})^4$. This strong suppression factor renders GWs from short-lived Goldstone modes entirely unobservable.

The situation is different for *long lived* Goldstone modes. There the suppression factor $(k\eta_*)^3$ is absent. Therefore, if the Goldstone modes remain massless until a time $\eta_{\rm fin}$, for modes with $k\eta_{\rm fin} \gtrsim 1$ the spectrum is scale invariant and the amplitude is given by

$$\Omega_{\rm GW}(k) \simeq \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4 \ , \tag{5.2}$$

which is marginally detectable with the experimental sensitivity of advanced LIGO or LISA and is well within the range of BBO for a GUT scale phase transition. The results for the long-lived source are summarized in Fig. 1.

If the Goldstone modes are still present at decoupling, $\eta_{\rm fin} \gtrsim \eta_{\rm dec}$, these GWs will also leave a signature in the cosmic microwave background where they lead to a scale-invariant contribution very similar to the one of global textures, i.e. a $N = 4$ global $\mathcal{O}(N)$ model [25].

Note that this new GW background from self-ordering fields after inflation (e.g. from hybrid preheating) has a power spectrum very similar to that coming from inflation, and therefore it may become important to disentangle both if they are present simultaneously, that is if the scale of inflation and that of symmetry breaking are related by parameters of order one, like in hybrid inflation.

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References

[1] M. Maggiore, *Gravitational Waves. Vol. 1: Theory and Experiments*, Oxford University Press (2007).

[2] M. Maggiore, Phys. Rept. **331**, 283 (2000); C. J. Hogan, arXiv:astro-ph/0608567; A. Buonanno, arXiv:0709.4682 [gr-qc].

[3] A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. D **45**, 4514 (1992); A. Kosowsky and M. S. Turner, Phys. Rev. D **47**, 4372 (1993) [arXiv:astro-ph/9211004]; A. Kosowsky and M. S. Turner, Phys. Rev. D **47**, 4372 (1993) [arXiv:astro-ph/9211004].

[4] M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D **49**, 2837 (1994) [arXiv:astro-ph/9310044].

[5] A. Kosowsky, A. Mack and T. Kahniashvili, Phys. Rev. D **66**, 024030 (2002) [arXiv:astro-ph/0111483]

[6] C. Caprini and R. Durrer, Phys. Rev. D **74**, 063521 (2006) [arXiv:astro-ph/0603476]; C. Grojean and G. Servant, Phys. Rev. D **75**, 043507 (2007) [arXiv:hep-ph/0607107]; T. Kahniashvili, A. Kosowsky, G. Gogoberidze, Y. Maravin, Phys. Rev. D **78**, 043003 (2008) [arXiv:0806.0293 [astro-ph]].

[7] C. Caprini, R. Durrer and G. Servant, Phys. Rev. D **77**, 124015 (2008) [arXiv:0711.2593 [astro-ph]].

[8] S. J. Huber and T. Konstandin, JCAP **0809**, 022 (2008) [arXiv:0806.1828 [hep-ph]].

[9] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. D **56** (1997) 653 [arXiv:hep-ph/9701423].

[10] J. Garcia-Bellido, *Preheating the Universe in hybrid inflation*, arXiv:hep-ph/9804205. Proceedings of 33rd Rencontres de Moriond: Fundamental Parameters in Cosmology, Les Arcs, France, 17-24 Jan 1998.

[11] R. Easther and E. A. Lim, JCAP **0604** (2006) 010 [arXiv:astro-ph/0601617].

[12] J. Garcia-Bellido, D. G. Figueroa, Phys. Rev. Lett. **98**, 061302 (2007) [arXiv:astro-ph/0701014].

[13] R. Easther, J. T. Giblin and E. A. Lim, Phys. Rev. Lett. **99**, 221301 (2007) [arXiv:astro-ph/0612294].

[14] J. Garcia-Bellido, D. G. Figueroa and A. Sastre, Phys. Rev. D **77**, 043517 (2008) [arXiv:0707.0839 [hep-ph]].

[15] J. F. Dufaux et al., Phys. Rev. D **76**, 123517 (2007) [arXiv:0707.0875].

[16] J. F. Dufaux, G. N. Felder, L. Kofman and O. Navros, JCAP **0903** (2009) 001 [arXiv:0812.2917 [astro-ph]].

[17] A. A. Starobinsky, JETP Lett. **30**, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)].

[18] J. Garcia-Bellido and A. D. Linde, Phys. Rev. D **57**, 6075 (1998) [arXiv:hep-ph/9711360].

[19] J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Phys. Rev. D **60**, 123504 (1999) [arXiv:hep-ph/9902449].

[20] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, Phys. Rev. Lett. **87**, 011601 (2001) [arXiv:hep-ph/0012142]; G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D **64**, 123517 (2001) [arXiv:hep-th/0106179].

[21] J. Garcia-Bellido and E. Ruiz Morales, Phys. Lett. B **536**, 193 (2002) [arXiv:hep-ph/0109230].

[22] J. Garcia-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, Phys. Rev. D **67**, 103501 (2003) [arXiv:hep-ph/0208228].

[23] A. Diaz-Gil, J. Garcia-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, Phys. Rev. Lett. **100**, 241301 (2008) [arXiv:0712.4263 [hep-ph]].

[24] R. Durrer and M. Kunz, Phys. Rev. **D55**, R4516 (1997) [arXiv:astro-ph/9612202].

[25] R. Durrer, M. Kunz and A. Melchiorri, Phys. Rev. **D59**, 123005 (1999) [arXiv:astro-ph/9811174].

[26] L. M. Krauss, Phys. Lett. B **284**, 229 (1992).

[27] K. Jones-Smith, L. M. Krauss and H. Mathur, Phys. Rev. Lett. **100** (2008) 131302 [arXiv:0712.0778 [astro-ph]].

[28] K. Jones-Smith, L. M. Krauss and H. Mathur, arXiv:0907.4857 [astro-ph.CO].

[29] B. Abbott et al. [LIGO Scientific Collaboration], Astrophys. J. **659**, 918 (2007) [arXiv:astro-ph/0608606];
URL: `http://www.ligo.caltech.edu/`.

[30] S. A. Hughes, arXiv:0711.0188 [gr-qc];
URL's: `http://lisa.esa.int` or `http://lisa.jpl.nasa.gov/`.

[31] V. Corbin and N. J. Cornish, Class. Quant. Grav. **23**, 2435 (2006);
G. M. Harry, P. Fritschel, D. A. Shaddock, W. Folkner and E. S. Phinney, Class. Quant. Grav. **23**, 4887 (2006);
URL: `http://universe.nasa.gov/new/program/bbo.html`.

[32] N. Turok and D.N. Spergel, Phys. Rev. Lett. **66**, 3093 (1991).

[33] R. Durrer, *Topological Defects in Cosmology*, in *New Astronomy Reviews* **43**, 111 (1999).

[34] R. Durrer, *The Cosmic Microwave Background*, Cambridge University Press (2008).

[35] S. Chongchitnan and G. Efstathiou, Phys. Rev. D **73** (2006) 083511 [arXiv:astro-ph/0602594].

[36] M. Maggiore, Phys. Rept. **331** (2000) 283 [arXiv:gr-qc/9909001].