Particle Creation in Pre-Big-Bang Cosmology: theory and observational consequences

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We present some phenomenological aspects of the pre-big-bang cosmological model inspired by the duality properties of string theory. In particular, assuming the spatial sections of the homogeneous background geometry to be isotropic, we discuss the quantum production of perturbations of the background fields (gravitons, dilatons, moduli fields), as well as the production of particles which do not contribute to the background, which we call "seeds". As such we consider the cases of electromagnetic and axionic seeds. We also discuss their possible observational consequences, for example, we study whether they can provide the origin of primordial galactic magnetic fields, and whether they can generate the initial fluctuations leading to the formation of large-scale structure and the measured cosmic microwave background anisotropies. We finally analyze axion and photon production in four dimensional anisotropic pre-big-bang cosmological models.

I. INTRODUCTION

The pre-big-bang scenario [1,2] (PBB) is a cosmological model inspired by the duality properties of string theory. It differs from conventional cosmological models based on general relativity by the presence of the dilaton field. In the PBB model, the initial state of the universe is the string perturbative vacuum, instead of a hot and dense state as predicted by the standard cosmological model. In the string (S) frame – in which weakly coupled strings move along geodesic surfaces [3] – the low-energy string effective action in D dimensions, neglecting both finite-size effects and loop corrections, can be written as

$$S = -\frac{1}{16\pi G_{\rm D}} \int d^D x \;\; \sqrt{|g_{\rm D}|} \;\; e^{-\phi} \times \left[R_{\rm D} + \partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} + V(\phi)\right] \; + \; S_{\rm M} \; , \tag{1.1}$$

where $G_{\rm D}$ is Newton's constant in D dimensions and $R_{\rm D}$ is the Ricci scalar of the D-dimensional space-time. In Eq. (1.1), $H_{\mu\nu\alpha}$ denotes the field strength of the two-index antisymmetric tensor $B_{\mu\nu}$ (i.e., $H = dB$), ϕ is the dilaton field and $V(\phi)$ is the dilaton potential. During the pre-big-bang era when the effective coupling $e^{\phi/2}$ is small, it is usually assumed that the dilaton potential ($V \sim \exp[-\exp(-\phi)]$) can be neglected. On the other hand, during the post-big-bang era the dilaton potential vanishes, leading to a constant massive dilaton sitting at the minimum. The action $S_{\rm M}$ describes "bulk" string matter satisfying the classical string equations of motion in a given background.

The low-energy string effective action, Eq. (1.1), leads to a set of cosmological equations for a homogeneous and spatially flat background which obey the "scale-factor duality" [1]: if the background fields are only time-dependent, and if $\{a_i(t), \phi(t); i = 1, ..., D-1\}$ denote the scale factors and dilaton of a given homogeneous exact solution with vanishing antisymmetric tensor, then the system $\{\tilde{a}_i(t), \tilde{\phi}(t); i = 1, ..., D-1\}$, obtained through the transformation in $(D-1)$ spatial dimensions:

$$a_i \to \tilde{a}_i = a_i^{-1} \;\; , \;\; \phi \to \tilde{\phi} = \phi - 2\sum_i \ln a_i \;\; , \tag{1.2}$$

is a new exact solution of the cosmological equations [1,4]. In the presence of matter, which can be approximated as a perfect fluid, a scale factor duality remains a symmetry of the solutions if we add the transformation law $p/\rho \to -p/\rho$ [1].

The solution of the string cosmology equations with the perturbative vacuum as initial condition, describes a phase of growing curvature and growing dilaton. It is called the "dilaton phase". Once the curvature H^2 (where $H = \dot{a}/a$; a dot stands for differentiation with respect to cosmic time t) reaches the string scale $\lambda_{\rm s}^{-1}$, the background enters the "string phase" during which higher derivative terms in the α' expansion become important. During this phase, the curvature scale is supposed to remain constant while the string coupling increases. At the beginning of the string phase, the string coupling can be arbitrarily small; this is one of the parameters of the model. At the end of the string phase, the dilaton approaches the strong coupling regime (the coupling constant is ~ 1), leading to the transition to the radiation-dominated era characterized by decelerated evolution and frozen dilaton field. The duration of the string phase is the second parameter of this cosmological model inspired by string theory. A complete description of the high curvature strong coupling regime and its transition to a radiation-dominated post-big-bang universe with frozen dilaton is still lacking. Some "toy models" and approximations have however been studied [5,6].

By combining the duality transformation with time-reversal symmetry ($t \to -t$), another symmetry of the model, we can always associate to any given decelerated expanding post-big-bang solution with decreasing curvature:

$$\dot{a} > 0 \;\; , \;\; \ddot{a} < 0 \;\; , \;\; \dot{H} < 0 \; , \tag{1.3}$$

an accelerated expanding pre-big-bang solution with growing curvature:

$$\dot{a} > 0 \quad , \quad \ddot{a} > 0 \quad , \quad \dot{H} > 0 \; . \tag{1.4}$$

Transforming this accelerating solution into the Einstein (E) frame — in which the dilaton coupling is absorbed into the metric by a conformal transformation, i.e., the graviton and dilaton kinetic terms are diagonalized — through the conformal $(D-1)$-dimensional rescaling [2]:

$$a \to a \; e^{-\phi/(D-2)} \quad , \quad dt \to dt \; e^{-\phi/(D-2)} \; , \tag{1.5}$$

the pre-big-bang solution transforms into a Friedmann universe with accelerated contraction and growing curvature

$$\dot{a} < 0 \quad , \quad \ddot{a} < 0 \quad , \quad \dot{H} < 0 \; . \tag{1.6}$$

Let us consider as an example [1], a spatially flat isotropic background in $D = 4$ space-time dimensions with vanishing antisymmetric tensor and dilaton potential and with non-zero bulk string matter. The scale factor duality maps the standard radiation-dominated solution with frozen dilaton

$$a \sim t^{1/2} \quad , \quad \phi = \text{const.} \quad , \quad \rho = 3p \quad , \quad t > 0 \tag{1.7}$$

into the pre-big-bang solution

$$a \sim (-t)^{-1/2} \; , \; \phi = -3\ln(-t) \; , \; \rho = -3p \quad , \quad t < 0 \tag{1.8}$$

which describes an accelerated expansion of the pole- or super-inflation type [7] with growing curvature and gravitational coupling.

The phenomenological drawbacks (i.e., horizon, flatness problems) of the standard cosmological model can be equally well solved, either by a sufficiently long era of super-inflation in the string frame, or an era of accelerated contraction in the Einstein frame. Thus, the choice of frame is just a matter of convenience, and the physical description of the PBB scenario is equivalent in the S and E frames [2]. In this paper we usually work in the string frame, otherwise we specify it.

The usual relations with the adiabatic decrease of temperature T:

$$\rho \sim 1/a^{\mathrm{D}} \sim T^{\mathrm{D}} \quad , \quad T \sim 1/a \tag{1.9}$$

in the radiation-dominated background, are mapped [2] to:

$$\rho \sim 1/a^{\mathrm{D}-2} \sim 1/T^{\mathrm{D}-2} \quad , \quad T \sim a \tag{1.10}$$

in the dual pre-big-bang background, where the temperature increases with the scale factor a. Going from the phase with scale factor a to the dual one with scale factor $1/a$, the temperature T remains unchanged, since we applied not only a duality transformation but also a time reversal, changing the range of cosmic time t from $[-\infty, 0]$ to $[0, \infty]$.

The PBB scenario attempts to describe the very early universe near the Planck scale, when general relativity breaks down. It should not be considered as an alternative to the conventional standard cosmological model supplied by inflation, but as a completion to it. Moreover, in full string theory the pre-big-bang solution is expected to be smoothly connected to a standard post-big-bang universe whose evolution is described by general relativity. One area of activity in string cosmology studies precisely this high curvature/high coupling phase which has to be described by string theory, either via corrections to the low curvature/low coupling equations or by some other means.

Another area of research deals with those observational consequences of the PBB model which are most probably independent of this transition and which distinguish the PBB model from other inflationary cosmologies. One then also wants to study how well different observables agree with present observational constraints. The cosmological implications of string theory, and more precisely, the comparison of the predictions of the PBB scenario with observations, will provide a test for string theory as a fundamental theory and fix some of the parameters of the PBB model. However, despite the attractive features of the PBB scenario, one should keep in mind that there still remain theoretical and phenomenological aspects which are not yet fully understood. Among the phenomenological aspects of the PBB scenario, vacuum fluctuations and the corresponding particle production are important issues and they will be discussed in this paper. In what follows, we calculate the spectra of the produced particles and indicate some possible observational consequences.

We consider phenomenological aspects of the pre-big-bang solutions derived from the low-energy string effective action. We decompose the spatial sections of the full D-dimensional space-time into a three-dimensional external expanding sub-manifold and a $(D-4)$-dimensional internal sub-manifold. The internal spatial dimensions shrink down to the final compactification scale, which is typically given by the fundamental length parameter of string theory.

At conformal time $\eta = -\eta_1$, when string corrections become important, we make an instantaneous transition from the pre-big-bang to the post-big-bang era where the internal dimensions as well as the dilaton are supposed to be frozen while the three external dimensions keep expanding like in an ordinary radiation-dominated Friedmann universe. A crucial but reasonable assumption, is that the spectra of the produced particles on scales much larger than η_1, are not influenced by the details of the transition from the pre- to the post-big-bang era. This has been verified numerically with some toy models for the pre- to post-big-bang transition [8]. In what follows, we neglect the effects of an intermediate string phase. More precisely, in our model the universe starts at the dilaton driven era and then enters the radiation-dominated era. It is possible that the intermediate string phase is long and affects a considerable region of the spectrum. Then, the spectra we calculate are still valid for all modes which exit the horizon before the string phase.

Our paper is organized as follows. In Section II we discuss the quantum creation of background perturbations (gravitons, dilatons and moduli fields). In the context of the PBB scenario, there is an accelerated shrinking of the event horizon during the super-inflationary phase [2], which ends when a maximal curvature scale is reached and higher-derivative terms can no longer be neglected. After an intermediate string phase, the universe enters [1–3] the decelerating, radiation-dominated phase. During the pre-big-bang phase the evolution, in either the Einstein or the string frame, is accelerated and the curvature as well as the dilaton field are growing. Therefore, the amplification of perturbations is more efficient at later times (higher frequencies). This generically implies blue spectra, in contrary to standard inflation where curvature remains essentially constant leading to flat spectra.

In Section III we discuss the quantum creation of fields which do not contribute to the background. We call them "seeds". Seeds are produced by the amplification of quantum fluctuations of fields which are present in string theory but are not part of the homogeneous background. In particular, we study electromagnetic (EM) seeds and their rôle for primordial galactic magnetic fields, and Kalb-Ramond axion seeds and their rôle for large-scale structure and the cosmic microwave background (CMB) anisotropies. Both cases are characteristic of the PBB scenario, since in the conventional picture based on general relativity, there is no Kalb-Ramond axion and electromagnetic perturbations cannot be excited due to their conformal coupling to the metric and the absence of a dilaton field.

Up to this point we assume the spatial section of the four-dimensional homogeneous background geometry to be isotropic, in other words there is a single scale factor. This assumption will be dropped in Section IV, where we analyze perturbations in four-dimensional anisotropic pre-big-bang models. In particular, we study the creation of axions and photons and compare the result with the isotropic case. In Section V we state our main conclusions.

Notation: Cosmic time is denoted by t, and conformal time by η; they are related by $t = \int a d\eta$, where a is the scale factor. Correspondingly, a dot stands for derivative w.r.t. cosmic time t, and a prime stands for derivative w.r.t. conformal time η. The scale factors are a (and b in the anisotropic case). The dilaton field is ϕ or φ, the modulus field is $\mathcal{B}$, and the axion field is σ. The spectral index of the perturbation spectrum is denoted by n. The gauge-invariant Bardeen potentials are Φ and Ψ. The critical density is $\rho_c = 3M_{\rm Pl}^2 H^2/(8\pi)$, where

$H = a'/a^2 = \dot{a}/a$. The transition scale in units of the Planck mass $M_{\rm Pl}$ is $g_1 = H_1/M_{\rm Pl}$. Since the universe is radiation dominated at time η_1, the fraction of the total (critical) energy density in radiation at a given time η is $\Omega_\gamma(\eta) = (H_1/H)^2(a_1/a)^4$. Latin indices i, j take values $1, 2, 3$, while capital letters A, B are equal to $1, ..., D-4$, where D stands for the dimensionality of the space-time. The subscripts E, S, A stand for Einstein, string and axion frames respectively.

II. PERTURBATIONS OF THE BACKGROUND

In this section we study perturbations that may be generated by parametric amplification of vacuum fluctuations, as the universe goes through the transition from the pre- to the post-big-bang era. In particular, we calculate the spectrum of quantum perturbations in the metric, dilaton and moduli fields produced in the classical PBB background. We assume the axion field not to contribute to the background energy density; of course quantum fluctuations of the axion field cannot be neglected and, as we show in the next section, they may even lead to the observed density perturbations.

During the pre-big-bang era, the universe goes through a phase of accelerated evolution with growing curvature scale, and decreasing co-moving Hubble length $|d\ln a(\eta)/d\eta|^{-1}$. Vacuum fluctuations in a given field which are initially, $\eta \to -\infty$, inside the Hubble scale exit at $\eta \sim -1/k$ and enter again in the subsequent radiation (or matter) era at $\eta \sim +1/k$. The time evolution of the background acts like a time-dependent potential and generically leads to particle creation. On scales k which are stretched to cosmologically large scales, this process can lead to classical perturbations of the gravitational, dilaton and moduli fields as in the usual inflationary scenario. The characteristics of the generated spectrum of perturbations depend on the evolution of the background and on the coupling of the fields.

We consider a D-dimensional space-time, which contains a four-dimensional homogeneous and isotropic external metric and a $(D-4) = m$-dimensional compactified internal metric. It is given by

$$ds^2_{\rm D} = -dt^2 + g_{ij}dx^i dx^j + \gamma_{AB}dx^A dx^B \ , \tag{2.1}$$

where $i, j = 1, 2, 3$, and $A, B = 1, ..., m = D-4$. In a four-dimensional, spatially homogeneous space-time the antisymmetric tensor field $H_{\mu\nu\alpha}$ has only one degree of freedom, which can be expressed by a pseudoscalar axion field σ. This "universal axion of string theory", is the four-dimensional dual of the Kalb-Ramond antisymmetric tensor field present in the low-energy string effective action [9]. In the four-dimensional external space-time, the effective dilaton and the antisymmetric tensor field are respectively [9]

$$\begin{aligned} \varphi &\equiv \phi - m\mathcal{B} \\ H^{abc} &\equiv e^{\varphi}\epsilon^{abcd}\nabla_d\sigma \ , \end{aligned} \tag{2.2}$$

where the modulus field $\exp(m\mathcal{B})$ determines the volume of the internal compactified space. Assuming the external four-dimensional space-time to be described by a Friedmann–Lemaître–Robertson–Walker (FLRW) metric whose space-like sections are flat and taking, for simplicity, also for the internal space an isotropic, homogeneous, flat, cylindrical ansatz, $\gamma_{AB} = \exp(2\mathcal{B})\delta_{AB}$, the dilaton-moduli-vacuum solutions (time-dependent dilaton and moduli fields, but constant axion field) are given in terms of conformal time η in the string frame by [9]

$$\begin{aligned} e^{\varphi} &\sim |\eta|^{\rm r} \sim |\eta|^{\frac{3\alpha-1}{1-\alpha}} \\ a &\sim |\eta|^{(1+{\rm r})/2} \sim |\eta|^{\frac{\alpha}{1-\alpha}} \\ e^{\mathcal{B}} &\sim |\eta|^{\rm s} \sim |\eta|^{\frac{\beta}{1-\alpha}} \ , \end{aligned} \tag{2.3}$$

with the Kasner constraint equation [9]

$$ {\rm r}^2 + 2m{\rm s}^2 = 3 \ , \qquad 3\alpha^2 + m\beta^2 = 1 \ . \tag{2.4}$$

The expressions in terms of r, s are equivalent to those in terms of α, β and both can be found in the literature. Note that $D = 4$ corresponds to $r_\pm = \pm\sqrt{3}$. The interesting solution describing an expanding external space is the one with $r < 0$ for which $1 + r < 0$ (or equivalently $\alpha < 0$) since $|\eta|$ is decreasing.

To study metric, scalar-dilaton and moduli fields perturbations, we define the external pump field P responsible for their amplification. For this, we identify for each perturbation the canonical variable ψ, which diagonalizes the perturbed action expanded up to second order [10]. By varying the perturbed action, we find that the Fourier modes $\psi_k(\eta)$ of each perturbation satisfy a decoupled, linear equation of the type

$$\psi_k'' + \left(k^2 - \frac{P''}{P}\right)\psi_k = 0 \ . \tag{2.5}$$

In the "sub-horizon regime" defined by $k^2P''/P \gg 1$, the solutions to Eq. (2.5) are simple plane-wave solutions; whereas in the "super-horizon regime" defined by $|k^2P''/P| \ll 1$, the general solution is given by

$$\psi_k(\eta) \simeq A_k P(\eta) + B_k \frac{1}{P(\eta)} \int^\eta \frac{d\tilde{\eta}}{P(\tilde{\eta})^2} \ , \quad |k^2P''/P| \ll 1 \ . \tag{2.6}$$

At the beginning of the pre-big-bang inflationary phase, each perturbation is well inside the horizon, $|k^2P''/P| \gg 1$. We normalize the plane-wave solutions of Eq. (2.5) in this regime to the vacuum fluctuation spectrum.

If P obeys a power-law, $P \propto (-\eta)^p$, the correctly normalized solution of Eq. (2.5) during the pre-big-bang phase can be written in terms of a Hankel function of second kind,

$$\psi_k = \eta^{1/2} H_\mu^{(2)}(|k\eta|) \ \ , \text{ with } \ \ \mu = \left|p - \frac{1}{2}\right| \ \ , \ \eta \leq -\eta_1 \ . \tag{2.7}$$

In the radiation-dominated era, we will find that the term P''/P vanishes, implying that the solutions of Eq. (2.5) are free plane-wave solutions

$$\psi_k = \frac{1}{\sqrt{k}}\left[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}\right] \ \ , \ \eta \geq -\eta_1. \tag{2.8}$$

The coefficients c_+, c_- are the Bogoliubov coefficients. Assuming that the universe goes from the pre-big-bang phase to the radiation-dominated era at the transition time $\eta = -\eta_1$, matching the pre-big-bang solution at $\eta = -\eta_1$ to the radiation era solution at $\eta = \eta_1$ determines the Bogoliubov coefficients.

The spectral energy density of produced particles, let us call them x, is related to the coefficient $c_-(k)$ by [10]

$$\rho_x(\omega) = \frac{d\rho_x}{d\log\omega} = \frac{\omega^4}{\pi^2}|c_-(\omega)|^2 \ . \tag{2.9}$$

The power spectrum Δ for energy density perturbations δx of a given field x is defined by

$$\Delta_{\delta x} \equiv \frac{k^3}{2\pi^2}|\delta x|^2 \tag{2.10}$$

and the spectral index n of the perturbation spectrum is defined by

$$n - 1 \equiv \frac{d\ln\Delta_{\delta x}}{d\ln k} \ . \tag{2.11}$$

The value $n = 1$ at second horizon crossing during the post-big-bang era characterizes a Harrison-Zel'dovich [11] or scale-invariant spectrum.

We study metric, dilaton and moduli fields perturbations in the Einstein frame which is conformally related to the string frame by

$$g_{ab}^{\rm E} = e^{-\varphi} g_{ab}^{\rm S} \ . \tag{2.12}$$

While strings are minimally coupled in the string frame, both dilaton and moduli fields are minimally coupled in the Einstein frame.

The scale factor in the Einstein frame, $a_{\rm E} = e^{-\varphi/2} a$, satisfies the Friedmann equation

$$\left(\frac{a'_{\rm E}}{a_{\rm E}}\right)^2 = \frac{8\pi G}{3}\left(\frac{1}{4}\varphi'^2 + \frac{m}{2}\mathcal{B}'^2\right) \ . \tag{2.13}$$

The fields φ and $\mathcal{B}$ behave like ordinary free, massless scalar fields in this background and satisfy the equations of motion [12]

$$\varphi'' + 2\frac{a'_{\rm E}}{a_{\rm E}}\varphi' = 0 \tag{2.14}$$

$$\mathcal{B}'' + 2\frac{a'_{\rm E}}{a_{\rm E}}\mathcal{B}' = 0 \ . \tag{2.15}$$

The background in the Einstein frame is contracting like $a_{\rm E} = \sqrt{|\eta/\eta_1|}$.

In the Einstein frame, first-order scalar and tensor perturbations of the metric are given, in the longitudinal gauge, by [12]

$$ds_{\rm E}^2 = a_{\rm E}^2(\eta)\left[-(1+2\Psi)d\eta^2 + \{(1+2\Phi)\delta_{ij} + h_{ij}\}\,dx^i dx^j\right] \ , \tag{2.16}$$

where Ψ, Φ are the gauge-invariant Bardeen potentials, h_{ij} denotes the tensor perturbation, and η denotes conformal time.

Metric perturbations are decomposed into scalar, vector and tensor modes, which to first order evolve independently. We disregard vector perturbations of the metric which, in the absence of seeds decay quickly during the radiation-dominated era. Moreover, in the spatially flat gauge, the evolution equations for the dilaton and moduli fields are, to first order, decoupled from the metric perturbations.

A. Scalar perturbations

The off-diagonal components of the perturbed Einstein's equations in the longitudinal gauge lead to the condition $\Phi = -\Psi$. The other components imply the decoupled equation [10]

$$\Psi_k'' + 6\frac{a'_{\rm E}}{a_{\rm E}}\Psi_k' + k^2\Psi_k = 0 \ , \tag{2.17}$$

for the Fourier mode Ψ_k, as well as the constraint equation

$$\Psi_k' + \frac{a'_{\rm E}}{a_{\rm E}}\Psi_k = \ 2\pi G\varphi'(\delta\varphi) + 4\pi G m\mathcal{B}'(\delta\mathcal{B}), \tag{2.18}$$

which relates the scalar metric perturbations to the dilaton and moduli field perturbations, $(\delta\varphi)$ and $(\delta\mathcal{B})$ respectively.

Analyzing the second order perturbed action, one finds that the variable v_k defined by

$$v_k = a\left[\frac{1}{2}(\delta\varphi)\varphi' + m(\delta\mathcal{B})\mathcal{B}' + \frac{\varphi'^2/2 + m\mathcal{B}'^2}{a'_{\rm E}/a_{\rm E}}\Psi_k\right]\frac{1}{\sqrt{\varphi'^2/2 + m\mathcal{B}'^2}} \tag{2.19}$$

is a canonical variable satisfying the equation of motion

$$v_k'' + (k^2 - \frac{a''_{\rm E}}{a_{\rm E}})v_k = 0 \ . \tag{2.20}$$

The variable v_k is related to the Bardeen potential Ψ_k by

$$\Psi_k = \frac{-4\pi G\sqrt{\varphi'^2/2 + m\mathcal{B}'^2}}{k^2}\left(\frac{v_k}{a}\right)' \ . \tag{2.21}$$

The correctly vacuum normalized solution of Eq. (2.20) is

$$v_k(\eta) = \eta^{1/2} H_0^{(2)}(k\eta) \ .$$

This leads to the following spectrum for the Bardeen potential [13]

$$k^{3/2}|\Psi_k| \simeq \frac{H_1}{M_{\rm Pl}}\frac{|k\eta_1|^{3/2}}{|k\eta|^2} \ , \tag{2.22}$$

where $H_1 \equiv H(\eta_1)$. From the above equation one might doubt whether linear perturbation is still valid when $\eta \to \eta_1$ since then the Bardeen potential can become very large. But one has to keep in mind that the Bardeen potential is not a measurable quantity and if one analyzes physical variables (*e.g.* $(C_{\mu\nu\alpha\beta})^2/(R_{\mu\nu\alpha\beta})^2$ where C and R denote the Weyl and Riemann curvatures respectively), one finds that the perturbation amplitudes remain small as long as $H_1 < M_{Pl}$. To illustrate this, we calculate the spectral energy density distribution associated with the Bardeen potential.

The Bardeen potential above is generated by some cosmic energy density perturbation satisfying the Poisson equation

$$4\pi G a^2 \rho_{\rm rad}\delta_x(k) = k^2\Psi_k \ , \tag{2.23}$$

and thus leading to a spectral distribution

$$k^3|\delta_x(k)|^2 \simeq g_1^2 (k\eta_1)^3 \tag{2.24}$$

which implies a spectral index $n = 4$. Here $g_1 = H_1/M_{\rm Pl}$ is the string coupling at the end of the pre-big-bang phase which is of the order of $10^{-3} \leq g_1 \leq 10^{-1}$. Since $|k\eta_1| < 1$, for all amplified frequencies, $\delta_x(k)$ is always a small contribution to the total energy density as long as $g_1 < 1$. A more detailed study of scalar metric perturbations can be found in Ref. [13].

A different but equivalent approach to the study of scalar perturbations in this background is the direct quantization of the dilaton and moduli fields which contribute the energy density perturbation δ_x. By consistency, this leads to the same spectral index, as we will now compute. We again work in the Einstein frame, in which both the dilaton and moduli fields evolve as minimally coupled massless fields. In the spatially flat gauge, the dilaton perturbations are decoupled from the axion perturbations and the evolution equation is Eq. (2.5) with

$$\psi_k = a_{\rm E}(\delta\varphi)_k \quad , \quad P = a_{\rm E} \ . \tag{2.25}$$

The moduli perturbations respectively obey, for each Fourier mode, the simple wave equation, Eq. (2.5), with

$$\psi_k = \sqrt{m}\, a_{\rm E}(\delta\mathcal{B})_k \quad , \quad P = a_{\rm E} \ ; \tag{2.26}$$

m stands for the number of internal compact dimensions.

Using $a_E \propto |\eta|^{1/2}$, the general solution of the evolution equation normalized to a vacuum fluctuation spectrum at $\eta \to -\infty$, can be written in terms of the zeroth Hankel function of the second kind as

$$\psi_k = \eta^{1/2} H_0^{(2)}(|k\eta|) \ , \ \eta \leq -\eta_1 \ . \tag{2.27}$$

In the radiation era, which follows the dilaton-driven era, one has instead free-plane wave solutions

$$\psi_k = \frac{1}{\sqrt{k}} \left[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}\right] \ . \tag{2.28}$$

The density parameter of produced dilatons per logarithmic frequency interval is determined by matching the solution (2.27) to (2.28) and using Eq. (2.9) (see also Ref. [14]):

$$\Omega_\varphi(\omega,\eta) = \frac{\rho(\omega)}{\rho_c} = \frac{1}{\rho_c}\frac{d\rho_\varphi}{d\log\omega} \simeq g_1^2 \left(\frac{\omega}{\omega_1}\right)^3 \left(\frac{H_1}{H}\right)^2 \left(\frac{a_1}{a}\right)^4 = g_1^2 \left(\frac{\omega}{\omega_1}\right)^3 \Omega_\gamma \ , \tag{2.29}$$

where $\rho(\omega)$ denotes the spectral energy density of produced dilatons, and $\omega_1 = k_1/a_1 = 1/(a_1|\eta_1|)$ represents the maximal amplified frequency.

More precisely, at second horizon crossing, the power spectrum of dilaton and moduli perturbations is [9]

$$\Delta_{\delta\varphi} \sim \frac{2}{\pi^3} H^2 (k\eta)^3 \ [\ln(k\eta)]^2$$
$$\Delta_{\delta\mathcal{B}} \sim \frac{2}{m\pi^3} H^2 (k\eta)^3 \ [\ln(k\eta)]^2 \ , \tag{2.30}$$

respectively. Thus, the amplitude of dilaton and moduli perturbations grows towards small scales, and for super-horizon scales it becomes large when $a_{\rm E}'/a_{\rm E}^2 \sim 1$.

Both the dilaton and moduli fields have steep blue perturbation spectra [9],

$$n_\varphi = n_\mathcal{B} = 4 \ . \tag{2.31}$$

According to Eq. (2.29), the contribution of these perturbations to the radiation density at the upper cutoff $\omega = \omega_1$ is of the order $\sim g_1^2 \ll 1$, on larger scales (lower frequencies) this contribution decreases like $(\omega/\omega_1)^3$ and hence it is completely negligible on all scales of cosmological interest. Note that the scale $\lambda_1 = 2\pi/\omega_1$ has expanded to about $\lambda_1(\eta_0) \sim 0.1$cm today. It is therefore unlikely that the dilaton and moduli quantum fluctuations from the pre-big-bang have left any characteristic observable signature in the present universe.

B. Tensor metric perturbations

For each Fourier mode, the evolution of tensor metric perturbations in the Einstein frame satisfies to lowest order Eq. (2.5) with

$$\psi_k = \frac{1}{\lambda_{\rm s}} a_{\rm E} (\delta h)_k \quad , \quad P = a_{\rm E} \ , \tag{2.32}$$

$\lambda_{\rm s} = \sqrt{\alpha'\hbar}$ denotes the short-distance cut-off of string theory and $a_E \propto |\eta|^{1/2}$ is the well-known expansion law for a scalar field dominated Friedmann universe, independent of the expansion/contraction of the internal dimensions. We note that in the S-frame, the string length parameter $\lambda_{\rm s}$ is constant. We have to divide δh by $\lambda_{\rm s}$ for dimensional reasons, the canonical field must have the dimensions of a scalar field. Like always, one finds ψ_k by writing the second order perturbation of the action in canonical form [15]. The solutions of the evolution equation are as in the last sub-section

$$\psi_k = \sqrt{|\eta|} H_0^{(2)}(k\eta) \quad \text{in the pre-big-bang,} \tag{2.33}$$
$$\psi_k = \frac{1}{\sqrt{k}} \left[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}\right] \ \ , \text{ in the post-big-bang} \tag{2.34}$$

Matching these solutions at the onset of the radiation-dominated era, $\eta = \eta_1$ determines the Bogoliubov coefficients. The magnitude of c_- gives the amplification of the gravitational waves with respect to the minimal vacuum fluctuation.

The spectrum depends only on the dynamics of the scale factor in the Einstein frame, which can be parameterized generically as $a_{\rm E}(\eta) = (-\eta)^{\gamma}$. The case of a dilaton and moduli background corresponds to $\gamma = 1/2$. For $\gamma \leq 1/2$, the co-moving amplitude $(\delta h)_k$ approaches a constant asymptotically ($|k\eta| << 1$) [15]. The perturbation amplitude $\delta_h(k)$ can be expressed in terms of the Hubble constant at horizon crossing (HC), defined by $|k\eta| \sim 1$, as [10]

$$|(\delta h)_k| \equiv k^{3/2}|h_k| \sim \left(\frac{H}{M_{\rm Pl}}\right)_{\rm HC} \sim g_1(k\eta_1)^{1+\gamma} = g_1(k\eta_1)^{3/2} \ , \tag{2.35}$$

where we have specified the dilaton-moduli background in the last equal sign. The amplitude remains constant in time. However, since higher-frequency modes cross the horizon at later times, therefore at a higher value of H if the curvature scale is growing, their amplitude is enhanced with respect to lower-frequency modes. This is different from the usual case of a scale-invariant spectrum, where the amplitude remains the same for all modes.

The amplitude of tensor perturbations over scales k^{-1} varies in time according to

$$|(\delta h)_k(\eta)| \sim g_1(k\eta_1)^{3/2} \ln|k\eta| \ . \tag{2.36}$$

A robust prediction of the PBB scenario is the finding that the spectrum of gravitational waves is characterized by rising amplitude with increasing frequency. The spectrum of primordial gravitational waves is steeply growing on short scales with a spectral index $n_{\rm T} = 3$, in contrast to the more conventional inflation models, for which $n_{\rm T} \lesssim 0$. If the duration of the intermediate string scale can be neglected, this steep blue spectrum leads to gravitational wave amplitudes of the order $|(\delta h)_0(f)|^2 \sim g_1^2(10^{11}Hz/f)^3$, far beyond the detection limits of any gravitational wave experiment in consideration. However, if the intermediate string scale is sufficiently long, the form of the amplitude of tensor perturbations can be modified (flattened) on small scales, and thus this conclusion can be altered (for a detailed discussion see [17]).

III. PERTURBATIONS OF FIELDS WHICH DO NOT CONTRIBUTE TO THE BACKGROUND: SEEDS

In the previous section we have discussed the perturbations obtained in the components which are also present in the background, the dilaton, the moduli and the gravitational field. Here we study perturbations of the fields which do not contribute to the background.

All quantum fields, even if their expectation value vanishes, exhibit quantum fluctuations. During an inflationary era, they stretch beyond the horizon scale and "freeze in" as classical non-vanishing inhomogeneous field configurations. Their contribution is in general second order (or higher) in the field perturbation, but it can nevertheless lead to appreciable perturbations in space-time.

Such second order perturbations can induce cosmic structure formation after the transition from the pre-big-bang to the radiation-dominated era in two ways. Either they may decay into dark matter particles and just leave their "imprint" as the initial condition for the dark matter and radiation perturbations. A model of this kind has been studied in Refs. [18,19]. Its main difference to ordinary cold dark matter (CDM)-like models is that the resulting perturbations are of isocurvature nature and that they do not obey Gaussian statistics [20]. Or, the perturbation may decouple from CDM and radiation and remain active as a "seed" which induces geometrical perturbations and perturbations in the radiation and CDM components, solely by gravitational interaction [21]. This is the possibility which has been studied in the case of axion field perturbations in Refs. [22–25].

As we shall point out in the next sub-section, second order perturbations can also remain in the form of large scale coherent magnetic fields and may contribute to the resolution of the problem of the origin of primordial magnetic fields [26].

A. Electromagnetic seeds and primordial magnetic fields

Let us first consider the photon field in the pre-big-bang universe. Since photons are conformally invariant they do not couple to a homogeneous and isotropic metric but they couple to the dilaton. Therefore, in contrast to ordinary inflation there is photon production in pre-big-bang inflation.

The "pump field" for photon perturbations is $P^{EM} = e^{-\varphi/2}$ with $\varphi = -2\gamma\log(-\eta/\eta_1)$, according to Eq. (2.3), $\gamma = -3\alpha/2(1-\alpha)$. Setting $\psi = e^{-\varphi/2}A_\mu$, the photon equation of motion in the radiation gauge, $A_0 = 0$ and $\partial^i A_i = 0$ in k-space reduces to

$$\psi_k'' + \left[k^2 - \frac{\gamma(\gamma-1)}{\eta^2}\right]\psi_k = 0. \tag{3.1}$$

The general solution of this equation is a linear combination of the Hankel functions $\eta^{1/2}H_\mu^{(1)}(k\eta)$ and $\eta^{1/2}H_\mu^{(2)}(k\eta)$ with $\mu = |\gamma - 1/2| = |1+2\alpha|/2(1-\alpha)$. At very early times, $\eta \ll -1/k$, the field is supposed to be in the in-coming vacuum state,

$$\psi_k = \sqrt{\frac{2}{\pi k}}e^{-k\eta} \ .$$

Therefore, the correctly normalized initial vacuum fluctuation spectrum, can be written in terms of the Hankel function of the second kind,

$$\psi_k = |\eta|^{1/2}H_\mu^{(2)}(k\eta) \ , \qquad \eta < -\eta_1 \ . \tag{3.2}$$

It describes an in-coming vacuum which, as the perturbation becomes super horizon ($-k\eta < 1$), becomes a non-trivial classical field configuration.

In the radiation era the dilaton is constant and ψ obeys an ordinary wave equation with solution

$$\psi_k = \frac{1}{\sqrt{k}}\left[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}\right] \ , \qquad \eta > \eta_1 \tag{3.3}$$

Using Eq. (3.2) as initial condition on super-horizon scales, $k\eta \ll 1$, and for $\eta \gg \eta_1$, we find

$$c_\pm = \pm c(k)e^{\pm ik\eta} \ , \qquad \psi_k = \frac{c(k)}{\sqrt{k}}\sin k(\eta - \eta_1) \ , \qquad |c(k)| \simeq (k/k_1)^{-\mu-1/2} \ , \tag{3.4}$$

where $k_1 = 1/|\eta_1|$ represents the maximal amplified frequency (higher-frequency modes are not amplified). According to Eq. (2.9) the associated energy-density distribution of the produced photons is

$$\frac{d\rho(k)}{d\log k} \simeq \left(\frac{k}{a}\right)^4 |c_-(k)|^2 \simeq \left(\frac{k_1}{a}\right)^4\left(\frac{k}{k_1}\right)^{3-2\mu} \ , \quad k < k_1 \ , \quad \mu < 3/2 \ , \tag{3.5}$$

where we require $\mu < 3/2$ (which is always satisfied for $-1 \le \alpha \le 0$) to avoid photon overproduction which would destroy the homogeneity of the classical background. The amplitude $c(k)$ has been estimated modulo numerical factors of order 1. At large times, $\eta \gg |\eta_1|$, we thus obtain in string cosmology, a cosmic background of electromagnetic fluctuations which is always blue, $n \ge 1.5$. This is too rapidly decaying to be of cosmological relevance *e.g.* as seeds for the observed large scale magnetic fields (see next section).

This conclusion is modified if there is a long intermediate string phase. During such a string phase, photon seeds are characterized by a rather flat spectrum, $\gamma \le 2$, and could provide the long-sought origin of the galactic magnetic fields [27].

The amplified fluctuations satisfy stochastic correlation functions as a consequence of their quantum origin.

In the radiation era we therefore have the following spectrum of electromagnetic perturbations

$$A_i(\mathbf{k},\eta) = \frac{c_i(\mathbf{k})}{\sqrt{k}} \sin k\eta \ , \qquad k_i A_i = 0 \ , \qquad A_0 = 0 \, . \tag{3.6}$$

A_i is a Gaussian random variable which obeys the stochastic average condition:

$$\langle A_i(\mathbf{k}) A_j^*(\mathbf{k}') \rangle = \frac{(2\pi)^3}{2} \delta^3(k-k') \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) |\mathbf{A}(\mathbf{k},\eta)|^2 \ . \tag{3.7}$$

The above condition has been normalized in such a way that

$$\sum_i \langle A_i(\mathbf{k}) A_i^*(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(k-k') \, |\mathbf{A}(\mathbf{k},\eta)|^2 \ . \tag{3.8}$$

Taking into account that the electric component of the stochastic background is rapidly dissipated due to the high conductivity of the cosmic plasma [28], only the magnetic field survives. Setting $B_i(k) = i\epsilon_{ijl} k_j A_l(k)$, the condition (3.7) implies

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{k}') \rangle = \frac{(2\pi)^3}{2} \delta^3(k-k') \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) b^2(k,\eta) \ , \tag{3.9}$$

where

$$b^2(\mathbf{k},\eta) = k^2 \, |\mathbf{A}(\mathbf{k},\eta)|^2 \, (a_1/a)^4 = k \, |\mathbf{c}(\mathbf{k})|^2 \sin^2 k\eta \ . \tag{3.10}$$

Here we have used that, within the magneto-hydrodynamic limit, magnetic fields in a Friedmann universe just conserve the flux per unit area and thus scale like $1/a^2$ (see, e.g. Ref. [29]).

In a process of photon production, the coefficient $|\mathbf{c}(\mathbf{k})|^2$ represents the Bogoliubov coefficient [10] fixing the average photon number density, $\langle n(k) \rangle$, and is linked to the spectral energy distribution by

$$\frac{d\rho(k)}{d\log k} = \left(\frac{k}{a}\right)^4 \frac{\langle n(k) \rangle}{\pi^2} \simeq \left(\frac{k}{a}\right)^4 \frac{|\mathbf{c}(\mathbf{k})|^2}{\pi^2} \ . \tag{3.11}$$

The spectrum $|\mathbf{c}(\mathbf{k})|^2$ is given by

$$|\mathbf{c}(\mathbf{k})|^2 = \begin{cases} (k/k_1)^{-2\mu-1} \ , & k \le k_1, \quad \mu \le 3/2 \\ 0 \ , & k > k_1 \ . \end{cases} \tag{3.12}$$

At first sight one might think that for $\mu = 3/2$ this also induces a scale-invariant (Harrison-Zel'dovich) spectrum of metric perturbations, but as it has been shown in Ref. [22] this is not the case. The reason is precisely the conformal coupling of photons.

The result (3.10) can be used to constrain (k_1, γ). From the CMB anisotropies induced by the gravitational coupling of the magnetic field one obtains [30]

$$B_1 < 7.9 \times 10^{-6} e^{3n} \ ,$$

where B_1 is the magnetic field amplitude on $\lambda = 0.1 h^{-1}$Mpc and n is the spectral index of the magnetic field spectrum. In our case $n = -2\mu$ and (see Ref. [30])

$$B_1^2 \simeq \frac{4}{(2\pi)^5} \lambda^{-4} (k_1 \lambda)^{2\mu+1}$$
$$\log(B_1) \simeq 41.2 + 24.5\mu \ ,$$

where we have used $\lambda = 0.1 h^{-1}$Mpc and $k_1 = 10^{18}\mathrm{GeV}/z_1$ with $z_1 = T_1/T_0 = 10^{18}\mathrm{GeV}/(2.5 \times 10^{-4}\mathrm{eV}) = 4 \times 10^{30}$.

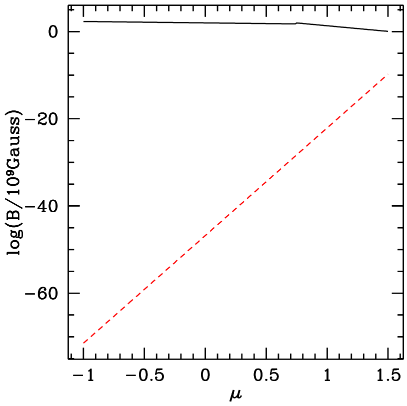

FIG. 1. The magnetic field induced in pre-big-bang cosmology is compared with the limit from CMB anisotropies. For all admitted values of the spectral index μ the induced field (solid line) is well below the limit (dashed line).

B. Axion seeds, large scale structure and CMB anisotropies

The Kalb-Ramond (KR) axion evolves as a massless minimally coupled field in the axion (A) frame, related to the Einstein frame by the conformal transformation

$$g^{\rm A}_{\alpha\beta} = e^{-2\varphi} g^{\rm E}_{\alpha\beta} \ . \tag{3.13}$$

We will use the axion frame to study Kalb-Ramond axion perturbations. Defining

$$\psi_A = a e^{\varphi/2} \sigma \equiv a_A \sigma \ , \tag{3.14}$$

we are led to the canonical equation

$$\psi_k'' + \left(k^2 - \frac{a_A''}{a_A}\right)\psi_k = 0 \ , \tag{3.15}$$

very similar to Eq. (3.1). The same procedure as in the electromagnetic case then leads to the spectrum (3.5) with $\mu = |r|$, where r parameterizes the three-dimensional axion scale factor as $a_A(\eta) \sim \eta^{r+1/2}$. For $r = -3/2$, in particular, the axion metric describes a de Sitter inflationary expansion, and the energy density of a massless KR axion background has a flat spectral distribution, $d\rho/d\log k \simeq (k_1/a)^4$, as first noted in Ref. [31]. The value of r depends on the number and on the kinematics of the internal dimensions, and the value $-3/2$ can be obtained, in particular, for a ten-dimensional background with special symmetries [16]. In the axion case, however, the low frequency tail of the spectrum is further affected by the radiation $\to$ matter transition, as the axion pump field a_A is not a constant (unlike the dilaton) in the matter-dominated era, where $a_A = a \propto \eta^2$.

In the radiation era, i.e. for $\eta_1 < \eta < \eta_{eq}$, the effective potential a_A''/a_A is vanishing, as $\varphi = const.$ and $a_A = a \sim \eta$, and ψ is given by the plane-wave solution (3.3). In the final matter-dominated era, i.e. for $\eta > \eta_{eq}$, we have $a \sim \eta^2$, and $a_A''/a_A = 2/\eta^2$. The plane-wave solution is still valid for modes with $k > k_{eq} = \eta_{eq}^{-1}$, which are unaffected by the last transitions. Modes with $k < k_{eq}$ feel instead the effect of the potential in the matter era, and the general solution of Eq. (3.15), for those modes, can be written as

$$\psi_k(\eta) = \frac{\sqrt{k\eta}}{\sqrt{k}}\left(AH^{(2)}_{3/2} + BH^{(1)}_{3/2}\right) \qquad k < k_{eq} \ , \qquad \eta > \eta_{eq} \ . \tag{3.16}$$

The matching of the solutions at η_1 determines the coefficients $c_\pm(k)$ as in Eq. (3.4). The matching at η_{eq} gives

$$A+B \sim c(\mathbf{k})\,(k\eta_{eq})^{-1} \quad , \qquad\qquad A-B \sim c(\mathbf{k})\,(k\eta_{eq})^{2} \quad , \tag{3.17}$$

In the matter-dominated era, i.e. for $\eta > \eta_{eq}$, we can then approximate the produced stochastic axion background as follows:

$$\begin{aligned}
\sigma(\mathbf{k},\eta) &\simeq \frac{c(\mathbf{k})}{a\sqrt{k}}\sin k\eta \quad , && k > k_{eq} \ , \\
&\simeq \frac{c(\mathbf{k})}{a\sqrt{k}}\left(\frac{k}{k_{eq}}\right)^{-1}(k\eta)^2 \ , && k < k_{eq} \quad , \quad k\eta < 1 \ , \\
&\simeq \frac{c(\mathbf{k})}{a\sqrt{k}}\left(\frac{k}{k_{eq}}\right)^{-1} \quad , && k < k_{eq} \quad , \quad k\eta > 1 \ .
\end{aligned} \tag{3.18}$$

The correlation functions for the various components of the stress tensor

$$T_\mu^\nu = \partial_\mu\sigma\partial^\nu\sigma - \frac{1}{2}\delta_\mu^\nu\,(\partial_\alpha\sigma)^2 \tag{3.19}$$

of massless KR axions can be computed by exploiting the stochastic average conditions of the Gaussian variables σ, σ' and $\sigma_j = \partial_j\sigma$:

$$\begin{aligned}
&\langle\sigma(\mathbf{k})\sigma^*(\mathbf{k}')\rangle = (2\pi)^3\delta^3(k-k')\Sigma_1(\mathbf{k},\eta) \ , \\
&\langle\sigma'(\mathbf{k})\sigma'^*(\mathbf{k}')\rangle = (2\pi)^3\delta^3(k-k')\Sigma_2(\mathbf{k},\eta) \ , \\
&\langle\sigma_i(\mathbf{k})\sigma_j^*(\mathbf{k}')\rangle = k_ik_j(2\pi)^3\delta^3(k-k')\Sigma_1(\mathbf{k},\eta) \ , \\
&\langle\sigma_j(\mathbf{k})\sigma'^*(\mathbf{k}')\rangle = -\langle\sigma'(\mathbf{k})\sigma_j^*(\mathbf{k}')\rangle = ik_j(2\pi)^3\delta^3(k-k')\Sigma_3(\mathbf{k},\eta) \ ;
\end{aligned} \tag{3.20}$$

the explicit form of $\Sigma_1, \Sigma_2, \Sigma_3$ can be found in Ref. [22]. For example for the axion energy density we find

$$\rho_\sigma = \frac{1}{2a^2}\left[\dot\sigma^2 + (\partial_i\sigma)^2\right] \ , \tag{3.21}$$

which leads to the spectrum

$$\begin{aligned}
k^3\langle|\rho_\sigma|^2\rangle = \frac{2k^3}{(2a^2)^2}\int_0^{k_1}\frac{d^3p}{(2\pi)^3}\Big[&\Sigma_2(\mathbf{p})\Sigma_2(\mathbf{k}-\mathbf{p}) + |\mathbf{p}\cdot(\mathbf{k}-\mathbf{p})|^2\,\Sigma_1(\mathbf{p})\Sigma_1(\mathbf{k}-\mathbf{p}) \\
&- 2\mathbf{p}\cdot(\mathbf{k}-\mathbf{p})\Sigma_3(\mathbf{p})\Sigma_3(\mathbf{k}-\mathbf{p})\Big] \ .
\end{aligned} \tag{3.22}$$

Similarly the other components of the energy-momentum tensor can be expressed in terms of Σ_1, Σ_1 and Σ_3 (see Ref. [25]). Simple approximations for the functions Σ_i can be found in Ref. [22].

The spectrum of $\langle|\rho_\sigma|^2\rangle$ on scales $k \ll k_1$, mainly depends on the behavior of the integral in Eq. (3.22). If this integral converges for $k=0$, then we always obtain a white noise spectrum $\langle|\rho_\sigma|^2\rangle =$ const.; this is the case for $\mu < 3/4$. Only if the integral diverges for $k=0$ we can get a non-trivial spectrum (with $0 < n \le 1$) for $\langle|\rho_\sigma|^2\rangle$.

Using linear perturbations of Einstein's equations one can determine the induced geometrical and dark matter perturbations. For the induced Bardeen potentials one can derive (after a lengthy calculation, see Ref. [22]) the following approximate expression, which is valid on large-scales ($k\eta \ll 1$) during the matter-dominated era ($\eta > \eta_{eq}$):

$$k^{3/2}\,|\Psi-\Phi| \simeq \begin{cases} g_1^2\Omega_\gamma(\eta)(\omega/\omega_1)^{-1/2}(\omega_1/H)^{-2}\left[1+\xi(\omega_{eq}/\omega_1)^2(\omega_1/H)^{2\mu+1/2}\right] \ , & \mu \le 3/4 \ , \\ g_1^2\Omega_\gamma(\eta)(\omega/\omega_1)^{-1/2}(\omega_{eq}/\omega_1)^2(\omega_1/H)^{2\mu-3/2} \ , & 3/4 \le \mu \le 3/2 \ , \end{cases} \tag{3.23}$$

where $\Omega_\gamma(\eta) = (H_1/H)^2(a_1/a)^4$ is the density parameter of the background radiation in the post-big-bang era, $\omega_1 = k_1/a$ is the redshifted string scale and ξ is a number of order unity [22]. Note that $H_1 \simeq \omega_1(\eta_1)$. The second case of Eq. (3.23) can also be written as

$$k^{3/2}\,|\Psi-\Phi| \simeq \; g_1^2\Omega_\gamma(\eta_0)(\eta_0/\eta_{eq})^2(k\eta)^{2\mu-7/2}(k/k_1)^{3-2\mu} \quad , \; 3/4 \le \mu \le 3/2 \; . \tag{3.24}$$

The above approximations are valid on super-horizon scales, $k\eta < 1$. On sub-horizon scales the "seed" energy-momentum tensor decays quickly due to the oscillatory behavior of the solution ψ_k, and the gravitational potential remains constant. On sub-horizon scales we therefore expect

$$k^{3/2}\,|\Psi-\Phi| \simeq g_1^2\Omega_\gamma(\eta_0)(\eta_0/\eta_{eq})^2(k/k_1)^{3-2\mu} \quad , \quad 3/4 \le \mu \le 3/2 \; . \tag{3.25}$$

This spectrum is scale-invariant, $k^3\,|\Psi-\Phi|^2$ =const. $\propto k^{n-1}$, for $\mu = 3/2$. Hence, massless Kalb-Ramond axion seeds with $\mu = 3/2$ can induce a Harrison-Zel'dovich spectrum of geometrical perturbations as it has been observed by the DMR experiment aboard the COBE satellite [33]. For $\mu < 3/2$ one obtains tilted spectra with $0 \le n \le 1$ where $n = 0$ for $\mu \le 3/4$.

For massive Kalb-Ramond axions the situation is qualitatively different if the axion mass is such that all super-horizon modes at the time of decoupling are already non-relativistic, $k/a < m$. In this case the contribution to temperature anisotropies is controlled by the axion mass, and a slightly blue spectrum is still compatible with the experimental constraints, provided the axion mass lies inside an appropriate ultra-light mass window with an upper limit of $10^{-17}eV$ [22]. This upper limit is rather low, thus if KR axions are heavier than $10^{-17}eV$, then they are incompatible with the data. However, if one considers more complicated cosmological backgrounds, it turns out that the bounds on the axion mass can be relaxed [34]. In particular, if one allows for an axion spectrum which grows monotonically with frequency, but with a frequency-dependent slope, then the non-relativistic KR axions can have a mass up to the $100MeV$ range [34]. This can be for example realized if the accelerated pre-big-bang evolution consists of at least two distinct eras [34].

To do a more precise numerical calculation, we have to write down the perturbed Einstein and matter equations which are of the form

$$\mathcal{D}_k X_k = \mathcal{S}_k \; , \tag{3.26}$$

where X_k is a long vector containing all the background perturbation variables, like the a_{lm}'s of the CMB anisotropies, the dark matter density fluctuation, the peculiar velocity potential etc., $\mathcal{D}$ is a linear ordinary differential operator and $\mathcal{S}_k$ is given by the energy-momentum tensor of the "seed", the axion in this case. The generic solution of this equation is of the form

$$X(\mathbf{k},\eta_0) = \int_{\eta_{in}}^{\eta_0} \mathcal{G}(\mathbf{k},\eta_0,\eta)\mathcal{S}(\mathbf{k},\eta)d\eta. \tag{3.27}$$

We want to determine power spectra or, more generally, quadratic expectation values which are then given by

$$\langle X_i(\mathbf{k},\eta_0)X_j(\mathbf{k},\eta_0)^*\rangle = \int_{\eta_{in}}^{\eta_0}\int_{\eta_{in}}^{\eta_0} \mathcal{G}_{il}(\eta_0,\eta)\mathcal{G}^*_{jm}(\eta_0,\eta')\langle\mathcal{S}_l(\eta)\mathcal{S}^*_m(\eta')\rangle d\eta d\eta'. \tag{3.28}$$

(Sums over double indices are understood.)

We therefore have to compute the *unequal time correlators*, $\langle\mathcal{S}_l(\eta)\mathcal{S}^*_m(\eta')\rangle$, of the seed energy-momentum tensor. This problem can, in general, be solved by an eigenvector expansion method [35,36]. The numerical solution for the CMB anisotropies obtained in Ref. [25] is shown in Fig. 2 below. The spectral index is chosen to be slightly blue, $\mu = 1.425$, to fit the data better. The cosmological parameters chosen for the plot are $\Omega_\Lambda = 0.85$, $\Omega_m = 0.4$, $h = 0.65$ and $\Omega_b h^2 = 0.02$. We compare the numerically obtained spectrum with the COBE [37] measurement and recent CMB anisotropy data, Boomerang98 [38] and Maxima [39].

Axionic seeds lead to isocurvature perturbations. To fit the observed position of the first acoustic peak it therefore requires a closed universe. The values adopted for Fig. 2 are also in agreement with the recent supernovae results which indicate an accelerating universe [40].

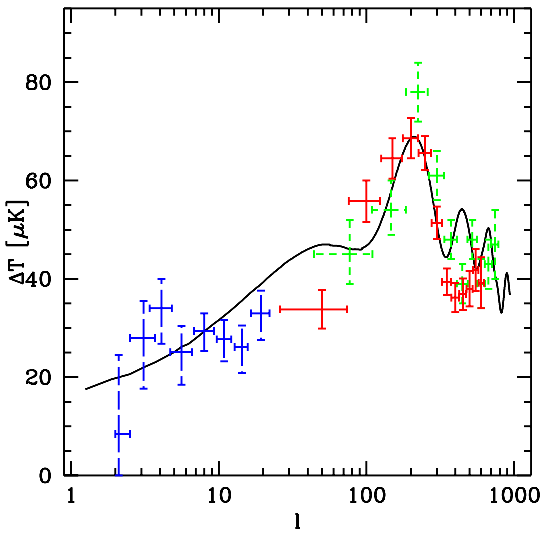

FIG. 2. The CMB anisotropies obtained from axionic seeds are compared with observations. The data used are the COBE data (dashed) on large scales, and the Boomerang98 data (solid) as well as the Maxima-1 data (dashed) on intermediate scales.

We consider this an important result which is detailed in Ref. [25]: a slightly blue spectrum of isocurvature perturbations in a closed universe with considerable cosmological constant can fit present CMB data. Also the dark matter power spectrum is in qualitative agreement with, *e.g.* the APM data.

IV. PERTURBATIONS IN AN ANISOTROPIC PRE-BIG-BANG MODEL

Recently there has been some evidence that the dilaton driven inflationary stage is generically spatially anisotropic. In Ref. [41] it has been proposed that the universe starts as a bath of stochastic gravitational and dilatonic waves. Some of these will be strong enough and collapse due to self gravity forming a black hole. A collapsing solution in the Einstein frame corresponds to an expanding solution in the string frame. Each of these black holes can be interpreted as a pre-big-bang bubble pinching off the spacetime and creating a new universe. However, the solution close to the singularity inside a black hole is generically described by a Kasner metric which is spatially homogeneous over some region but not necessarily isotropic. In Ref. [42] a different realization of this proposal was given. Here pre-big-bang bubbles are created in the interaction region of two colliding plane waves. In the interaction region a singularity is generically formed which again described by an anisotropic Kasner metric.

The fate of global anisotropy in spatially homogeneous backgrounds during the dilaton driven inflationary stage was studied in Ref. [43]. It was found that these persist and are *not* inflated away as in the usual potential-dominated inflationary scenarios. Therefore it seems to be important to investigate if there are any observational imprints from an anisotropic dilaton driven inflationary stage. We will consider axionic and electromagnetic density perturbation spectra. In order to simplify, only axi-symmetric backgrounds will be considered. This, of course, means to impose an additional symmetry, but on the other hand makes the whole problem more tractable.

A. Axion production

Let us consider a four-dimensional PBB cosmological model with the line element

$$ds^2 = a^2(\eta)d\eta^2 - a^2(\eta)dx^2 - b^2(\eta)dy^2 - b^2(\eta)dz^2. \tag{4.1}$$

Here it is assumed that the internal dimensions are frozen and only the dilaton remains as a dynamical field whereas the form fields are supposed to have zero field strength in the background.

The background evolution is then given by [1]

$$a(\eta) = \left[-\frac{\eta}{\eta_1}\right]^{\frac{\alpha}{1-\alpha}}, \qquad b(\eta) = \left[-\frac{\eta}{\eta_1}\right]^{\frac{\beta}{1-\alpha}} , \tag{4.2}$$

and the evolution of the dilaton,

$$\phi(\eta) = \left(\frac{\alpha + 2\beta - 1}{1-\alpha}\right) \log\left[-\frac{\eta}{\eta_1}\right] , \tag{4.3}$$

with α and β satisfying the Kasner condition

$$\alpha^2 + 2\beta^2 = 1.$$

Pre-big-bang inflationary solutions are described by $\alpha < 0$ and $\beta < 0$. Only these will be considered here.

The evolution of the Kalb-Ramond axion field σ defined in Eq. (2.2) in Fourier space is determined by [44] [45]

$$\psi_k'' + \left[k_L^2 + k_T^2\frac{a^2}{b^2} - \frac{P''}{P}\right]\psi_k = 0 , \tag{4.4}$$

where the canonical field ψ is defined as $\psi = e^{\phi/2}b\sigma$, the pump field is $P = e^{\phi/2}b$, k_L denotes the modulus of the comoving longitudinal momentum and $k_T = \sqrt{k_y^2 + k_z^2}$ is the modulus of the transverse momentum. The anisotropy of the background space-time is reflected in the asymmetry between the longitudinal and transverse momenta. For comparison, the case of axion production in an isotropic space-time has been discussed in Sec. IIIB.

Here also we assume that there is an instantaneous transition from the end of the dilaton phase at $\eta = -\eta_1$ to the radiation-dominated FLRW post big-bang era.

The aim is to calculate the spectral energy density of the axionic inhomogeneities $(d\rho_\sigma/d\log\omega)$ as they re-enter the horizon during the isotropic radiation-dominated era, after being amplified during the anisotropic dilaton-dominated epoch. Inserting the expressions for the scale factors (cf. (4.2)) and the dilaton (cf. (4.3)) in Eq. (4.4) one obtains [44] [45]

$$\psi_k'' + \left[k_L^2 + k_T^2\left(-\frac{\eta}{\eta_1}\right)^\gamma - \frac{\mu^2 - 1/4}{\eta^2}\right]\psi_k = 0 , \tag{4.5}$$

where

$$\gamma = \frac{2(\alpha-\beta)}{1-\alpha} \qquad p = \frac{\alpha + 4\beta - 1}{2(1-\alpha)} ,$$
$$2\mu = |\, 2p - 1\,| = 2 - \frac{4\beta}{1-\alpha}.$$

We first consider the case $\gamma < 0$. Then, the term k_L^2 always dominates the bracket in Eq. (4.5) at very early time $\eta \to -\infty$. There is no known exact analytic solution of Eq. (4.5). Therefore to make progress two limiting cases will be discussed [45]:

Case (I): The modulus of the longitudinal momentum, k_L, always dominates until $\eta^2 < 1/k_L^2$ at which point the $1/\eta^2$-term comes to dominate. This is equivalent to the condition

$$k_T < k_L(k_1/k_L)^{-\gamma/2} . \tag{4.6}$$

Case (II): At some conformal time $\eta = \eta_T < -\eta_1$ the modulus of the transverse momentum, k_T, comes to dominate over k_L, but the mode is still well within the horizon, i.e. $\sqrt{k_L^2 + k_T^2(-\eta_T/\eta_1)^\gamma} > \eta_T^{-2}$. Then Eq. (4.5) implies

$$\eta_T = -\eta_1 \left(\frac{k_L}{k_T}\right)^{2/\gamma} . \tag{4.7}$$

Let us examine each of these two cases separately.
Case (I): Neglecting the k_T^2 term leads to a Bessel equation which during the pre-big-bang era has the solution, for $\eta \leq -\eta_1$

$$\psi_k^{PBB}(k,\eta) = \sqrt{\frac{|k_L\eta|}{k_L}} H_\mu^{(2)}(k_L\eta) . \tag{4.8}$$

During the radiation-dominated FLRW post-big-bang stage, the solution, for $\eta \geq -\eta_1$, is

$$\psi_k^{RD} = \frac{1}{\sqrt{k}}\left[c_+ e^{-ik(\eta+\eta_1)} + c_- e^{ik(\eta+\eta_1)}\right] . \tag{4.9}$$

Matching the two solutions Eq. (4.8) and Eq. (4.9) determines the frequency mixing coefficient c_- which allows to calculate the occupation numbers of produced axions. The spectral energy density of the produced axions is

$$\rho_L = \frac{d\rho_\sigma}{d\log\omega} \simeq \frac{\omega^4}{\pi^2}|c_-|^2 , \tag{4.10}$$

which with

$$c_- = -\frac{1}{2\pi}\sqrt{\frac{1}{(k\eta_1)(k_L\eta_1)^{2\mu}}} \tag{4.11}$$

yields to

$$\rho_L(\omega,s) \simeq \frac{\omega_1^4}{2\pi^3} s^{-2\mu}\left(\frac{\omega}{\omega_1}\right)^{3-2\mu} , \tag{4.12}$$

where $s = k_L/k$. Note that in the special case $\mu = 3/2$, corresponding to $\alpha = -7/9$, $\beta = -4/9$, a flat spectrum is obtained.
Case (II): Assuming that the k_T-term comes to dominate before the perturbation becomes super-horizon Eq. (4.5) may be approximated by

$$\psi_k'' + \left(k_L^2 + k_T^2\left[-\frac{\eta}{\eta_1}\right]^\gamma\right)\psi_k = 0. \tag{4.13}$$

An approximate solution of Eq. (4.13) is

$$\psi \simeq \frac{\exp(i\eta\sqrt{k_L^2 + q^2(-\eta/\eta_1)^\gamma k_T^2})}{\sqrt{\pi/2}[k_L^2 + (-\eta/\eta_1)^\gamma \ , k_T^2]^{1/4}} \tag{4.14}$$

where $q = 1/(1+\gamma/2) = (1-\alpha)/(1-\beta)$. This represents the in-coming vacuum solution.

In Fig. 3, we present numerical solutions of Eq. (4.13) and compare them with the approximate analytical solution Eq. (4.14) for different values of s and γ. As the comparison shows the approximate analytic solution Eq. (4.14) is very good for small as well as large values of s (cf. Fig. 3).

At conformal time $\eta = \eta_T$, the transverse momentum k_T becomes dominant over the k_L term and finally the η^{-2} dominates. After time $\eta = \eta_T$ Eq. (4.5) can be approximated by

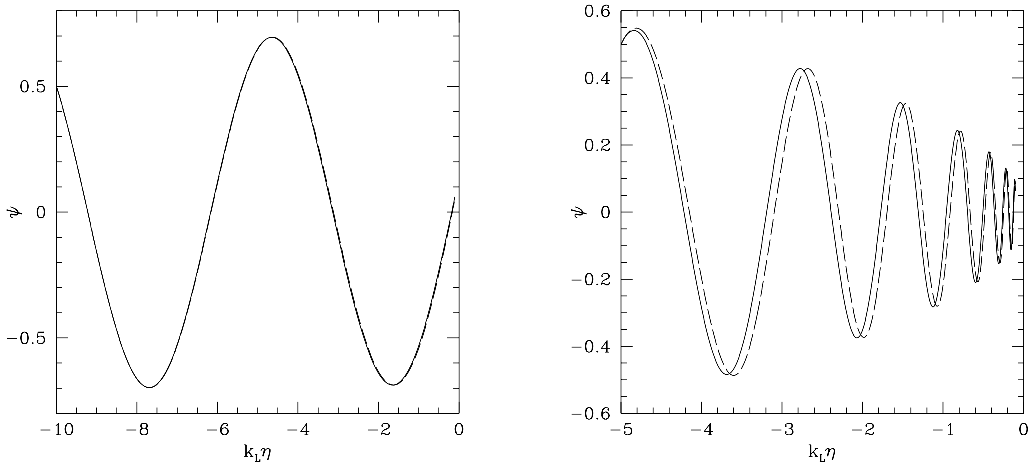

FIG. 3. Comparison of the numerical and approximate solution for $s = 0.93$, $\gamma = -0.5$ (left) and $s = 0.1$, $\gamma = -1.9$ (right). The solid line represents the numerical solution and the dashed one the approximate solution.

$$\psi_k'' + \left[k_T^2 \left(-\frac{\eta}{\eta_1}\right)^{\gamma} - \frac{\mu^2 - 1/4}{\eta^2} \ , \right] \psi_k = 0 \tag{4.15}$$

which has the general solution

$$\begin{aligned} \psi_k = & \, c_T^{(1)} \sqrt{|k_T \eta|} H_{\mu q}^{(1)} \left(|k_T \eta| q \left[\frac{-\eta}{\eta_1}\right]^{\gamma/2} \right) \\ & -i c_T^{(2)} \sqrt{|k_T \eta|} H_{\mu q}^{(2)} \left(|k_T \eta| q \left[\frac{-\eta}{\eta_1}\right]^{\gamma/2} \right) \ , \end{aligned} \tag{4.16}$$

$H_{\mu q}^{(1)}$ and $H_{\mu q}^{(2)}$ are Hankel functions of the first and second kind of order μq. For large $k_T|\eta|$ the second term just corresponds to the approximate solution, Eq. (4.14). Therefore matching these solutions, the coefficients are found to be

$$c_T^{(1)} = 0 \quad , \quad c_T^{(2)} = \frac{i}{\sqrt{k_T}} \ . \tag{4.17}$$

For super-horizon perturbations ($|k_T \eta_1| \ll 1$), matching the physical fields at the transition time $\eta = -\eta_1$, for $|k_T \eta_1| \ll 1$ (i.e., from the end of dilaton-driven era to the beginning of the radiation-dominated post-big-bang universe) one obtains the Bogoliubov coefficient c_-:

$$|c_-|^2 = \left[\frac{\Gamma^2(\mu q)}{4\pi^2} 2^{2\mu q} \left(\frac{3}{2} - \mu q\right)^2\right] \left(\frac{k_T}{k_L}\right)^{-2\mu q} s^{-2\mu q} \left(\frac{\omega}{\omega_1}\right)^{-1-2\mu q} \ . \tag{4.18}$$

The energy density of the produced Kalb-Ramond axions, in the case where the transverse momentum k_T is dominant, is

$$\rho_T(\omega, s) = \left[\frac{\Gamma^2(\mu q)}{4\pi^2} 2^{2\mu q} \left(\frac{3}{2} - \mu q\right)^2\right] \frac{1}{\pi^2} \left(\frac{k}{k_T}\right)^{2\mu q} \omega_1^{1+2\mu q} \omega^{3-2\mu q} \ . \tag{4.19}$$

Therefore, in summary, we obtain

$$\rho(\omega, s) \simeq \frac{\omega_1^4}{2\pi^3} \begin{cases} s^{-2\mu} \left(\frac{\omega}{\omega_1}\right)^{3-2\mu} & \text{if } k_T < k_L (k_1/k_L)^{-\gamma/2} \\ (1-s^2)^{-\mu q} \left(\frac{\omega}{\omega_1}\right)^{3-2\mu q} & \text{else.} \end{cases} \tag{4.20}$$

This can be expressed in terms of $\Omega_\gamma(\eta) = (H_1/H)^2(a_1/a)^4$, i.e. of the fraction of critical energy density in radiation at a given time η, and of $g_1 = H_1/M_{\rm Pl}$, the transition scale in units of the Planck mass $M_{\rm Pl}$, as

$$\Omega_\sigma(\omega, s, \eta) \simeq g_1^2 \Omega_\gamma(\eta) \begin{cases} (1-s^2)^{-\mu q} \left(\frac{\omega}{\omega_1}\right)^{3-2\mu q} & \text{if } s \le s_c(\omega) \\ s^{-2\mu} \left(\frac{\omega}{\omega_1}\right)^{3-2\mu} & \text{if } s \ge s_c(\omega) \end{cases} \tag{4.21}$$

For a given value of ω, the parameter s_c is determined by the equation

$$\sqrt{1-s_c^2} = s_c^{1+\gamma/2} \left(\frac{\omega}{\omega_1}\right)^{\gamma/2} . \tag{4.22}$$

To estimate the total energy density per logarithmic frequency value one has to integrate the axion density $\Omega_\sigma(\omega, \eta, s)$ over s. Using $d^3k = 4\pi k^2 ds \wedge dk$ results in the following expression for $\Omega_\sigma(\omega, \eta)$:

$$\begin{aligned} \Omega_\sigma(\omega, \eta) &= \int \Omega_\sigma(\omega, \eta, s) ds \\ &\simeq g_1^2 \Omega_\gamma \left[\left(\frac{\omega}{\omega_1}\right)^{3-2\mu q} \int_0^{s_c(\omega)} (1-s^2)^{-\mu q} ds + \left(\frac{\omega}{\omega_1}\right)^{3-2\mu} \int_{s_c(\omega)}^1 s^{-2\mu} ds \right] . \end{aligned} \tag{4.23}$$

Using that $\omega < \omega_1$ the value of s_c can be approximated in the two cases $\gamma < 0$ and $\gamma > 0$ as follows,

$$s_c \simeq \left(\frac{\omega}{\omega_1}\right)^{q-1} \qquad \text{if } \gamma < 0 \tag{4.24}$$

$$1 - s_c^2 \simeq \left(\frac{\omega}{\omega_1}\right)^{\frac{2}{q}-2} \qquad \text{if } \gamma > 0 \tag{4.25}$$

with $q = 1 + /(1+\gamma/2)$. Strictly speaking, our analysis is only valid for $\gamma < 0$, but the result remains correct also for $\gamma > 0$, as one can easily check using our approximation Eq. (4.14), which in this case then describes the in-coming vacuum solution. Thus, the integrals in Eq. (4.23) can be approximated, to give

$$\Omega_\sigma(\omega, \eta) \simeq g_1^2 \Omega_\gamma(\eta) \left(\frac{\omega}{\omega_1}\right)^n , \tag{4.26}$$

where

$$n = 2 + q - 2\mu q = \frac{1+\alpha+2\beta}{1-\beta} \qquad \text{if } \alpha < \beta$$
$$n = 1 + \frac{2}{q} - 2\mu = \frac{1+\alpha+2\beta}{1-\alpha} \qquad \text{if } \alpha > \beta .$$

Since, $\alpha^2 + 2\beta^2 = 1$ and for PBB inflation $\alpha, \beta \le 0$, it follows that $\alpha + 2\beta \le -1$. This implies that the spectrum is generically red and only the degenerate case with two static and one inflating dimensions, $(\alpha = -1, \beta = 0)$ yields a flat spectrum. The spectral index is relatively close to the isotropic value, $n_{iso} = 3 - 2\sqrt{3} \sim -0.46$, for all reasonable values of α and β (see Fig. 4). In conclusion, the anisotropic expansion has little influence on the overall axion production, while as we have seen in the previous section, extra dimensions can yield generically a flat spectrum of axions.

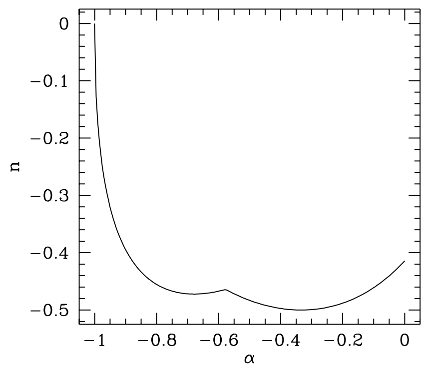

FIG. 4. The spectral index n of the axion perturbations is shown as a function of the Kasner exponent α. Except in the one-dimensional limit $\alpha \to -1$, the spectral index is always very close to the isotropic result, the value for $\alpha = 1/\sqrt{3}$.

B. Photon production

An interesting aspect of photon production in anisotropic space-times is that even without the coupling of the dilaton to the Maxwell part of the action, there is photon production since anisotropic space-times are not conformally flat [46]. Again, we restrict our analysis to the axi-symmetric case. In the string frame Maxwell's equations read

$$\partial_\mu (e^{-\phi}\sqrt{-g}F^{\mu\nu}) = 0 \ , \tag{4.27}$$

$$\partial_\mu (\sqrt{-g} \star F^{\mu\nu}) = 0 \ , \tag{4.28}$$

Setting $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the electromagnetic gauge potential, we solve Eq. (4.28) identically. For a metric $ds^2 = -dt^2 + a_i^2(dx^i)^2$ Eq. (4.27) leads in momentum-space to:

$$\sum_{i=1}^{3} a_i^{-2} i k_i [\partial_0 A_i - i k_i A_0] = 0 \ , \tag{4.29}$$

$$-\partial_0[e^{-\phi}\sqrt{-g}a_m^{-2}(\partial_0 A_m - i k_m A_0)] + e^{-\phi}\sqrt{-g}a_m^{-2}\sum_{n=1}^{3} a_n^{-2} i k_n [i k_n A_m - i k_m A_n] = 0 \ , \tag{4.30}$$

where $m = 1, 2, 3$ (no sum).

We impose the gauge conditions $A_0 \equiv 0$ and $\underline{k}.\underline{A} = 0 \Rightarrow \sum_i g^{ii} k_i A_i = 0 \Rightarrow \sum_i a_i^{-2} k_i A_i = 0$ which corresponds to the radiation gauge. Using the axi-symmetric metric Eq. (4.1), this gauge condition together with Eq. (4.29) imply the constraint,

$$A_L \equiv 0 \quad \text{or} \quad k_L \equiv 0 \ , \tag{4.31}$$

where the index L denotes the longitudinal direction. The gauge condition then reduces to $\vec{k_T} \cdot \vec{A_T} = 0$, where $\vec{v_T}$ denotes the two-dimensional vector in the $(y-z)$ plane. $\vec{A_T}$ hence has one degree of freedom, normal to $\vec{k_T}$, which we simply denote by A_T. Due to the breaking of spherical symmetry the longitudinal (A_L) and transverse (A_T) degrees of freedom obey different equations of motion. Introducing the canonical fields $\psi_L \equiv e^{-\phi/2}(b/a)A_L$ and $\psi_T \equiv e^{-\phi/2}A_T$, Eq. (4.30) leads to

$$\psi_L'' + \left[k_L^2 + \left(-\frac{\eta}{\eta_1}\right)^\gamma k_T^2 - \frac{\lambda_L}{\eta^2}\right]\psi_L = 0 \ , \tag{4.32}$$

$$\psi_T'' + \left[k_L^2 + \left(-\frac{\eta}{\eta_1}\right)^\gamma k_T^2 - \frac{\lambda_T}{\eta^2}\right]\psi_T = 0 \ , \tag{4.33}$$

where

$$\gamma \equiv \frac{2(\alpha-\beta)}{(1-\alpha)} \quad , \quad \lambda_L \equiv \frac{(3\alpha-1)(1+\alpha)}{4(1-\alpha)^2} \quad , \quad \lambda_T \equiv \frac{(-\alpha+2\beta+1)(\alpha+2\beta-1)}{4(1-\alpha)^2} \ . \tag{4.34}$$

There are two cases to discuss. The above set of equations has to be solved for either (a) $\psi_L \equiv 0$ or (b) $k_L \equiv 0$. Case (a) reduces the system to just one independent equation. Therefore, in this case, we can simply adapt the discussion of the previous section on axion production in an anisotropic background. In case (b) there are two independent equations. However, since $k_L \equiv 0$ they reduce to Bessel equations which are exactly solvable.

- **Case (a):** $\Psi_L \equiv 0$
  - **Case (I):** If the longitudinal momentum (k_L) dominates as long as the perturbation is sub-horizon, the spectral energy density in the radiation era is obtained exactly like in the axionic case:

$$\rho_L(\omega,s) \simeq 2\frac{\omega_1^4}{2\pi^3}s^{-2\mu_T}\left(\frac{\omega}{\omega_1}\right)^{3-2\mu_T} \ , \tag{4.35}$$

    where $\mu_T \equiv (\frac{1}{4}+\lambda_T)^{\frac{1}{2}} = |\beta/(1-\alpha)|$. A flat spectrum is recovered for $\mu_T = 3/2$. Using the Kasner constraint this corresponds to a positive value of α, namely $\alpha = 7/11, \beta = \pm 6/11$. Hence in this case, PBB inflation only takes place in the transverse but not in the longitudinal direction.
  - **Case (II)** If the transverse momentum (k_T) comes to dominate on sub-horizon scales, the in-coming vacuum solution is approximately given by Eq. (4.14). The solution for super-horizon modes during the dilaton-driven inflationary stage is again given by Eqs. (4.16),(4.17) with $\mu = \mu_T$. The matching of the gauge potential and its first derivatives at the transition from the dilaton-driven era to the radiation-dominated FLRW universe at $\eta = -\eta_1$ determines the Bogoliubov coefficient c_-. Recalling that $A_T = e^{\phi/2}\psi_T$, it is found that for $|k_T\eta_1| \ll 1$,

$$|c_-|^2 = \frac{1}{4\pi^2}\Gamma(\mu_T q)^2\left(\frac{q}{2}\right)^{-2\mu_T q}(k_T\eta_1)^{-2\mu_T q}(k\eta_1)^{-1}\left[\left(\mu_T - \frac{\beta}{1-\alpha}\right)^2 + \mathcal{O}(k\eta_1)^2\right] \ , \tag{4.36}$$

    where $q = (1-\alpha)/(1-\beta)$ and $\mu_T = |\beta/(1-\alpha)|$. The first term in the last square bracket vanishes for positive β. However, only solutions with $\beta < 0$ are really of interest here since they describe PBB inflationary expansion. In summary, for $\beta < 0$ the spectral energy density of the produced photons is

$$\rho_T(\omega,s) \simeq 2\frac{\mu_T^2}{\pi^4}\omega_1^4\left[\Gamma(\mu_T q)\right]^2\left(\frac{q}{2}\right)^{-2\mu_T q}(1-s^2)^{-\mu_T q}\left(\frac{\omega}{\omega_1}\right)^{3-2\mu_T q} \ . \tag{4.37}$$

    Thus the density parameter Ω_{em} is

$$\Omega_{em}(\omega,s,\eta) \simeq 2g_1^2\Omega_\gamma(\eta)\begin{cases} (1-s^2)^{-\mu_T q}\left(\frac{\omega}{\omega_1}\right)^{3-2\mu_T q} & \text{if } s \le s_c(\omega) \\ s^{-2\mu_T}\left(\frac{\omega}{\omega_1}\right)^{3-2\mu_T} & \text{if } s \ge s_c(\omega) \ . \end{cases} \tag{4.38}$$

    In order to estimate the total energy density per logarithmic frequency interval, $\Omega_{em}(\omega,s,\eta)$ has to be integrated over s. The value of s_c is determined as before by Eq. (4.22). Carrying out the integration, making use of Eqs. (4.24) and (4.25), the density parameter of the produced electromagnetic spectrum reads

$$\Omega_{em}(\omega,\eta) \sim g_1^2 \Omega_\gamma(\eta) \left(\frac{\omega}{\omega_1}\right)^n \ , \tag{4.39}$$

with the spectral index n,

$$n = \begin{cases} 2 + q - 2\mu_T q = \frac{3-\alpha}{1-\beta} & \text{if } \alpha < \beta \\ 1 + \frac{2}{q} - 2\mu_T = \frac{3-\alpha}{1-\alpha} & \text{if } \alpha > \beta \ . \end{cases} \tag{4.40}$$

In contrast to the axion case, the photon spectrum is always blue. For $\alpha = \beta = -1/\sqrt{3}$ the isotropic spectral index, $n_{iso} = 4 - \sqrt{3}$ is recovered [47]. In Fig. 5 the spectral index n is shown as a function of α.

- **Case (b)**: $k_L \equiv 0$

  This case is very particular since it describes the production of photons with wavenumber confined to the symmetry plane.

  Since $\psi_L \not\equiv 0$, there are two independent equations

$$\psi_L'' + \left[\left(-\frac{\eta}{\eta_1}\right)^\gamma k_T^2 - \frac{\lambda_L}{\eta^2}\right]\psi_L = 0 \ , \tag{4.41}$$

$$\psi_T'' + \left[\left(-\frac{\eta}{\eta_1}\right)^\gamma k_T^2 - \frac{\lambda_T}{\eta^2}\right]\psi_T = 0 \ . \tag{4.42}$$

These equations can be solved in terms of Bessel functions. However, the overall discussion follows closely that of case (a) (II). The in-coming vacuum solution is again given by Eq. (4.14) which in this case is an exact solution during the pre-big-bang phase. The matching procedure is the same for A_T, but note that the solution for ψ in the radiation-dominated era is also just a function of $k_T = k$. Performing a similar calculation as before for A_L, the Bogoliubov coefficient for A_L–photons is

$$|c_-^{(L)}|^2 = \frac{1}{4\pi^2}\left[\Gamma(\mu_L q)\right]^2 \left(\frac{q}{2}\right)^{-2\mu_L q} (k_T\eta_1)^{-2\mu_L q - 1} \left[\left(-\frac{\alpha}{1-\alpha} + \mu_L\right)^2 + \mathcal{O}(k_T\eta_1)^2\right] \ , \tag{4.43}$$

where $\lambda_L \equiv \mu_L^2 - 1/4$ implies $\mu_L = |\alpha/(1-\alpha)|$.

The spectral energy density, keeping in mind that this result holds just for $s = k_L/k = 0$, is

$$\rho \simeq \frac{\omega^4}{\pi^2}\left(|c_-^{(L)}|^2 + |c_-^{(T)}|^2\right)\delta(s) \ , \tag{4.44}$$

where $|c_-^{(T)}|^2$ is given by Eq. (4.36) with k replaced by k_T. Integration over directions then yields the density parameter

$$\Omega_{em}(\omega,\eta) \simeq g_1^2 \Omega_\gamma \left(\mathcal{N}_L \left(\frac{\omega}{\omega_1}\right)^{m_L} + \mathcal{N}_T \left(\frac{\omega}{\omega_1}\right)^{m_T}\right) \ , \tag{4.45}$$

where $\mathcal{N}_\bullet$ collects all the numerical factors of order unity in ρ and $|c_-^{(\bullet)}|^2$, respectively, and $m_\bullet$ are given by

$$m_L = \frac{3 + 2\alpha - 3\beta}{1-\beta}$$
$$m_T = \frac{3-\beta}{1-\beta}. \tag{4.46}$$

The "effective spectral index", the smaller of the two, $m_* = \min(m_L, m_T)$, is indicated in Fig. 5. Again, we always obtain blue spectra, if all dimensions are expanding, $\alpha, \beta < 0$.

In order to discuss the production of primordial magnetic fields it is useful to introduce a parameter $r(\omega)$ defined by [28]

$$r(\omega) \equiv \frac{\Omega_{em}}{\Omega_\gamma}. \tag{4.47}$$

Galaxies are endowed with magnetic fields of typical strength of order 10^{-6} G, coherent on a comoving scale of $\lambda_G \sim 10$ kpc. Assuming the existence of some kind of galactic dynamo r needs to be of order $r(\omega_G) \geq 10^{-34}$ [28].

This implies a constraint on the spectral index. With $\omega_G \simeq (10^{-2}\text{Mpc})^{-1} \simeq 10^{-36}\text{GeV}$ and $\omega_1 \simeq H_1 = g_1 M_{Pl} \simeq 10^{16} - 10^{18}\text{GeV}$, where $g_1 \simeq 10^{-3} - 10^{-1}$, this requires $n < 0.59$. This cannot be achieved in this anisotropic PBB model (cf. Fig. 5). Furthermore, the spectral index is minimal for the photons produced on an isotropic background, or for $k_L = 0$.

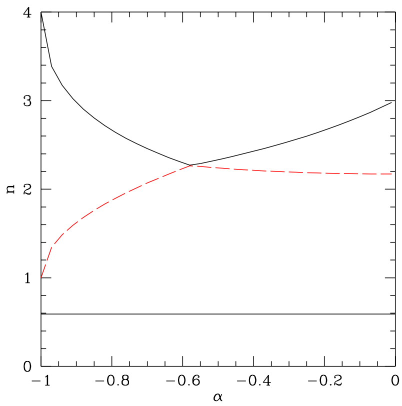

FIG. 5. The spectral index n for photon production in an anisotropic pre-big-bang (case (a) solid, case (b) dashed) and is shown as a function of the Kasner exponent α. To obtain a strong enough magnetic field for successive amplification by a galactic dynamo mechanism n has to lie below the line $n \sim 0.59$.

Clearly, in the isotropic case the contributions from cases (a) and (b) coïncide. Note however that in the anisotropic case photon production in the plane $k_L = 0$ dominates ($m_* < n$, flatter spectra) over the polarized photons with $A_L = 0$. The $k_L = 0$ photons are specific to the anisotropic, not conformally flat case and do lead to an enhancement of photon production in this case. Nevertheless, the effect of global anisotropies is not sufficient to produce the necessary primordial magnetic fields. Once again allowing for evolving extra dimensions or a long intermediate string phase, one can push the spectral index sufficiently down to create strong enough magnetic seed fields [27].

V. CONCLUSIONS

We have analyzed some phenomenological aspects induced by particle production in the PBB scenario, which is a particular cosmological model inspired by the duality properties of string theory. Assuming that the transition from the pre- to the post-big-bang era will not affect the observational consequences of the PBB model, we compare the theoretical observables with current observational data. In doing so, we provide a test for string theory as a fundamental theory and fix some of the parameters of the PBB model.

We first considered a D-dimensional space-time, containing a four-dimensional homogeneous and isotropic external metric and a $(D-4)$-dimensional compactified internal metric, with vanishing axion contribution. We study the evolution of perturbations that may be generated by the parametric amplification of vacuum fluctuations as the universe goes from the pre- to the post-big-bang era. We study scalar and tensor metric perturbations. We find that scalar as well as tensor perturbations have very similar amplitudes, and spectra which are growing towards large wave-numbers (i.e., blue spectra). A robust prediction of the PBB model is $n_{\rm T} = 3$, while standard inflationary models require $n_{\rm T} < 0$. Dilaton and moduli field perturbations have also steep blue spectra with a spectral index $n = 4$. This is of course very different from a scale-invariant Harrison-Zel'dovich spectrum with $n = 1$. These blue spectra, which are normalized to g_1^2 at the high frequency end, decay rapidly towards longer wavelengths and are completely negligible on cosmological scales.

Even if a field does not contribute to the background evolution, quantum fluctuations cannot be neglected. The induced energy density perturbation is then of second order in the field perturbation, but it can lead to appreciable perturbations in space-time geometry. Perturbations of fields which do not contribute to the background are refered to as "seeds". We analyze electromagnetic and Kalb-Ramond axion seeds, and study their rôle for the origin of primordial galactic magnetic fields, the large-scale structure and CMB anisotropies. We compute the stochastic fluctuations of the energy-momentum tensor of the seeds and determine their contribution to the multipole expansion of the temperature anisotropy. We find that electromagnetic perturbations lead to a "blue" power spectrum whose amplitude is fixed at the string scale; on larger scales, it decays too fast to produce primordial magnetic fields which may be amplified to the presently observed values. However, since the contribution of electromagnetic perturbations to the large-scale anisotropy is negligible, the COBE normalization does not impose any constraints to the production of seeds for galactic magnetic fields. Kalb-Ramond axions, which are either massless or have a mass up to $100MeV$ can lead to a flat or slightly blue spectrum, in reasonable agreement with current data. The actual value of the axion spectral index depends on the rate of contraction of the internal dimensions during the pre-big-bang era.

We also consider four-dimensional spatially flat anisotropic PBB cosmological models. Computing the energy spectra for massless Kalb-Ramond axions, we find that, when integrated over directions, this model leads to infra-red divergent spectra, as in the case of a four-dimensional spatially flat isotropic model. Similarly, analyzing photon production in this background, we find that the obtained blue spectra do not differ significantly from the isotropic case. Therefore, also the four-dimensional anisotropic pre-big-bang models suffer from the unphysical red axion spectrum and the blue photon spectrum found in the four-dimensional isotropic pre-big-bang models.

The analysis presented here is valid for modes which exit the horizon before the string phase. However, if the intermediate string phase is sufficiently long, the spectra can be affected on sufficiently large scales which may be of interest for cosmology. Some examples of spectra from significantly long string phase have been explored in Refs. [34,27,16]).

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