The Oscillating Universe: an Alternative to Inflation

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Abstract

The aim of this paper is to show, that the 'oscillating universe' is a viable alternative to inflation. We remind that this model provides a natural solution to the flatness or entropy and to the horizon problem of standard cosmology. We study the evolution of density perturbations and determine the power spectrum in a closed universe. The results lead to constraints of how a previous cycle might have looked like. We argue that most of the radiation entropy of the present universe may have originated from gravitational entropy produced in a previous cycle.

We show that measurements of the power spectrum on very large scales could in principle decide whether our universe is closed, flat or open.

1 Introduction

In a closed universe, the inevitable big crunch might actually be followed by a subsequent big bang. This idea is very old. It goes back to Lemaîtres Phoenix picture of 1933 [1]. As we shall remind especially younger readers, this 'oscillating universe' provides quite naturally a solution to the flatness or entropy and to the horizon problem. This was well known before the advent of inflationary models around 1980 in seminal papers by Starobinski, Guth, Linde and others [2]. Since then, it has been so completely forgotten, that we ourselves originally believed to be the first to study these ideas and older colleagues had to refer us to the original literature. However, it is not only inflation which let people forget about the oscillatory universe, but also an argument in essence due to Penrose [3] that black hole formation in a previous cycle leads to far too much entropy and thereby to an even more severe entropy problem in the opposite sense than the usual one.

It is the aim of this paper to show that under most realistic circumstances Penrose's conclusion need not be drawn. We shall see that some amount of entropy production due to gravitational clumping can just somewhat accelerate the growth of the maximal scale factor, $a_{\max}$, from one cycle to the next, without over producing entropy. This will lead us to the conclusion that the oscillating universe remains a viable alternative to inflation.

We consider this especially important since inflation has become some kind of 'cosmological dogma' during the last ten years, despite the fact that no inflationary scenario which solves the horizon and flatness problems and yields acceptably small density fluctuations has yet been constructed without substantial fine tuning (which may be protected by a symmetry and thus be 'technically natural'). Furthermore, many inflationary models are built upon the gravitational action of a cosmological constant, the most miraculous number in cosmology, which today is by a factor of about 10^{100} times smaller than what we would expect from particle physics [4]. A mechanism relying on such a completely mysterious number, to us, seems very unsatisfactory.

Besides solving the horizon and entropy problems, inflation generically predicts a scale invariant Harrison–Zel'dovich initial spectrum of fluctuations as it was observed by the DMR experiment on the COBE satellite [5]. This observations have been considered as great success of even 'proof' of inflation. However, also global topological defects [6] or cosmic strings [7], which can form during phase transitions in the early universe, naturally lead to a scale invariant spectrum of fluctuations but they cannot easily be reconciled with inflation.

These considerations prompted us to look for possibilities to solve the flatness and the horizon problem without invoking an inflationary period.

The basic picture which we work out in this paper is the following:
The first 'big bang', the first 3–dimensional closed universe, emerged from quantum fluctuation of some, e.g., string vacuum. Its duration was of the order of a Planck time. Due to some non thermal processes there was a small gain of entropy, $S_{in}^{(1)} < S_{end}^{(1)}$. The first big crunch triggered the formation of the next big bang who's entropy was slightly larger and therefore its duration was slightly longer, $S_{in}^{(1)} < S_{end}^{(1)} \leq S_{in}^{(2)}$. This process continues with ever longer cycles. We assume, that after a few Planck times during which the universe may have been in some quantum gravity or stringy state, we have a mainly classical, radiation dominated universe. With the exception of short periods during which matter and radiation fall briefly out of thermal equilibrium, the universe expands and contracts adiabatically. As long as the universe is radiation dominated no black holes can form. Only during a cycle with a long enough matter dominated era black hole formation is possible. Penrose's argument now goes as follows: During a matter dominated era (small) black holes can form. These finally, during the collapse phase, coalesce into one huge black hole which at the end contains the whole mass of the universe. Its entropy is thus given by

$$S_{bh} = (1/2)A_{bh}/G = 2\pi R_s^2/G = 8\pi G M_{bh}^2 \geq 10^{124}, \tag{1}$$

where we have set M_{bh} equal to the present mass within one Hubble volume which is of order $10^{23}M_{\odot}$ to obtain the last inequality. Clearly, already a significantly smaller mass would do, since the actual radiation entropy within the present Hubble radius is of the order of $S_{Hubble} \sim 10^{87}$, a discrepancy of nearly 40 orders of magnitude. In terms of entropy per baryon, this yields $\eta^{-1} = S/N_B \approx 10^{44}$ instead of the observed value $\eta^{-1} \approx 10^9$.

Is there a way out of this simple but disastrous conclusion?
The first and main objection is that the laws of black hole thermodynamics which rely heavily on Hawking radiation, hold only in asymptotically flat spacetimes. Or, at least, that the entropy of a black hole can be set equal to its area only for an observer outside the black hole itself. Therefore, the black hole entropy formula should only be adopted for black holes much smaller than the size of the universe. Let us therefore add only the entropy of black holes which are at least 10 times smaller than the curvature radius of the universe and neglect subsequent

growth of entropy due to the coalescence of these black holes. Of course, this rule is somewhat ad hoc, but as long as we have no clue of how to calculate in general the entropy of the gravitational field it seems to us a possible 'rule of thumb'. However, with this correction we gain only about a factor of 10 in the above entropy formula (1) and not the required factor of about 10^{38}. But there is an additional short come in the Penrose conclusion: The radiation entropy which we observe today is the entropy generated mainly during the previous cycle whose matter dominated epoch might have been much shorter, leading to significantly less clumping and thus much less gravitational entropy production.

Furthermore, as we shall see, it is not clear that structure forms via hierarchical clustering. In a pure radiation universe, it may well be that large black holes form first (if at all!) and the black hole entropy formula cannot be applied.

From these arguments it should be clear, that Penrose's objection to the oscillating universe does not have to be accepted and there may be ways out. Another possibility not investigated in this work is Israel's idea of mass inflation inside the horizon of black holes [8, 9]. There Israel et al. accept the black hole entropy formula, but argue that inside the black hole horizon mass inflation takes place such that the ratio η^{-1} gets reduced substantially.

The reminder of this paper is organized as follows: In the next section we give a brief review of the oscillatory universe. In the main part of this paper, section 3, we investigate cosmological perturbation theory in a closed universe and determine the evolution of a Harrison Zel'dovich initial spectrum in a purely radiation dominated universe and in a universe with an intermediate matter dominated epoch. We discuss how the final spectrum depends on the duration of the matter dominated epoch and we formulate a limit for the maximum radiation entropy of the previous cycle. The final section is devoted to our conclusions.

Notation: The scale factor of the Friedmann universe is denoted by a, we use the conformal time coordinate and the metric signature $(-,+,+,+)$, so that the Friedmann metric is given by

$$ds^2 = a^2(dt^2 - \gamma_{ij}dx^i dx^j) \ , \tag{2}$$

where γ_{ij} is the metric of the unit three sphere, e.g.,

$$\gamma_{ij}dx^i dx^j = d\chi^2 + \sin^2(\chi)(d\theta^2 + \sin^2\theta d\varphi^2) \ .$$

Cosmic time is denoted by τ, $\tau = \int^t a dt$.
We normalize the scale factor a such that the curvature of the spatial sections is equal to $1/a^2$.

2 Reminder to the oscillatory universe

Let us first explain how the oscillatory universe solves the flatness or entropy problem. To do this it is useful to state the problem in a somewhat different form: If the equation of state in a Friedmann universe satisfies the strong energy condition, $\rho + 3p > 0$, then ρ decays faster than $1/a^2$ and $\Omega = 1$ is the unstable 'initial' fix point of expansion. This means each Friedmann universe starts out at $t \approx$ a few $t_{\rm Planck}$ with $\Omega \approx 1$ and later deviates more and more from this value. The flatness problem can thus be stated as follows: How can it be, that our universe at its old age, $t \gg t_{\rm Planck}$, $T \ll T_{\rm Planck}$ still looks so young, $\Omega \sim 1$? This problem is easily solved in the oscillating universe as we shall now show.

The following arguments are due to Tolman [10]. Only a year after Lemaître first brought up his phoenix picture Tolman realized: Since the entropy of the next universal expansion can only be larger than the previous one, the maximum expansion factor of the next cycle, $a_{\rm max}$, is larger than the corresponding maximum of the previous cycle. Since the density parameter Ω starts deviating from 1 only when the scale factor a approaches $a_{\rm max}$, in the next cycle it will take longer until this happens. We consider now a cycle with a duration substantially longer than Planck time which has entered a radiation dominated phase. If relativistic matter is in thermal equilibrium (which we assume to be true most of the time) its energy density and entropy density are given by ($\hbar = c = k_{\rm Boltzmann} = 1$)

$$\rho = \frac{\pi^2}{30} N T^4 \tag{3}$$

$$s = \frac{2\pi^2}{45} N T^3 , \tag{4}$$

where N denotes the effective number of degrees of freedom (spin states). $N = N_b + (7/8)N_f$. Here N_b are bosonic degrees of freedom and N_f are fermionic degrees of freedom. Furthermore, from Friedmann's equation for a closed universe,

$$\left(\frac{\dot{a}}{a}\right)^2 + 1 = \frac{8\pi G}{3} a^2 \rho , \tag{5}$$

together with (3) and (4) one finds

$$\Omega - 1 = \frac{\rho - \rho_c}{\rho_c} = \frac{8\pi G \rho a^2 - 3(\dot{a}/a)^2}{3(\dot{a}/a)^2} = \frac{1}{G\left(\frac{4N\pi}{45}\right)^{1/3} S^{2/3} T^2 - 1} . \tag{6}$$

Here S is the total entropy of the universe, $S = 2\pi^2 a^3 s$. Therefore, the larger the total entropy S the smaller the deviation of Ω from the critical value 1 at a given temperature T, or the lower temperatures are required for a substantial increase of Ω. Expressing the maximal scale factor, $a_{\rm max}$, the age of the universe at maximal expansion, $\tau_{\rm max} = \tau(a_{\rm max})$ and the minimal temperature, $T_{\rm min} = T(a_{\rm max})$, in terms of the entropy also show that these values grow, respectively decrease with increasing entropy:

$$a_{\rm max} = \lambda_1 S^{2/3} \quad , \quad \lambda_1 = \left(\frac{45 G^3}{4\pi^7 N}\right)^{1/6} , \tag{7}$$

$$\tau_{\rm max} = \int^{t_{\rm max}} a dt = a_{\rm max} \quad , \quad t_{\rm max} = \pi/2 , \tag{8}$$

$$T_{\rm min} = \lambda_2 S^{-1/3} \quad , \quad \lambda_2 = \left(\frac{45}{4\pi G^3 N}\right)^{1/6} . \tag{9}$$

The time it takes for the density parameter to differ significantly from 1 is a substantial fraction of $\tau_{\max}$. Therefore, the universe 'looks young' for longer and longer times as the entropy increases cycle by cycle.

It is clear that in the oscillating universe also the horizon problem disappears since the age of the universe is not given approximately by the inverse Hubble time, which is the age of the present cycle, but the sum of the ages of all previous cycles has to be added, leading to a much larger age which might even be infinite. For this solution to be valid, it is important that correlations are not lost during a big crunch/big bang passage and that the behavior of particles or strings during this time is governed by a causal theory. In Appendix B, we explore the possibility that quantum gravity may effectively lead to an Euclidean region of spacetime close to the big crunch/big bang era. This example of a causal continuation from one cycle to the next is due to Ellis [11, 12]. The singularity in the metric induced by the signature change is very mild. We show how in this case geodesics can be continued through the crunch in a completely smooth manner.

3 Cosmological Perturbation Theory in a Closed Universe

In this section we first study the equations which govern the time evolution of radiation and matter density perturbations in a closed universe. We then determine the power spectrum, which we use to decide at which length scale the density perturbations might first lead to the formation of objects (e.g. galaxies or black holes). We finally use these results to argue how the entropy of the present cycle may have been generated.

3.1 Time Evolution of Density Perturbations

To describe the time evolution of density perturbations we use gauge invariant linear cosmological perturbation theory (see e.g. [13]). Assuming adiabatic perturbations and neglecting anisotropic stresses, the evolution of the gauge invariant density perturbation variable D is governed by the equation

$$\ddot{D} - (\nabla^2 + 3)c_s^2 D + (1 + 3c_s^2 - 6\omega)\left(\frac{\dot{a}}{a}\right)\dot{D} \tag{10}$$

$$-3\left\{\omega\left(\frac{\ddot{a}}{a}\right) - 3\left(\frac{\dot{a}}{a}\right)^2 (c_s^2 - \omega) + (1+\omega)\frac{4}{3}\pi G\rho a^2\right\} D = 0.$$

In a universe which consists of matter and radiation, $\omega = p/\rho = (1/3)(1+a/a_{eq})^{-1}$, $c_s^2 = \dot{p}/\dot{\rho} = (1/3)(1 + 3a/4a_{eq})^{-1}$, where c_s is the sound velocity. A dot indicates derivatives with respect to conformal time t and a_{eq} is the scale factor when $\rho_{rad} = \rho_{mat}$. Two cases of particular interest are dust ($\omega = c_s^2 = 0,\ a_{eq} = 0$) and radiation ($\omega = c_s^2 = 1/3,\ a_{eq} = \infty$).

Expanding D in terms of scaler harmonic functions on $\mathbf{S}^3$, as described in Appendix A, leads to the following equation for the gauge invariant density perturbation amplitude for the wavenumber $l \in \{0, 1, 2, ...\}$:

$$\ddot{D}_l(t) + (l(l+2) - 3)c_s^2 D_l(t) + (1 + 3c_s^2 - 6\omega)\left(\frac{\dot{a}}{a}\right)\dot{D}_l(t)$$
$$-3\left\{\omega\left(\frac{\ddot{a}}{a}\right) - 3\left(\frac{\dot{a}}{a}\right)^2 (c_s^2 - \omega) + (1+\omega)\frac{4}{3}\pi G\rho a^2\right\} D_l(t) = 0. \quad (11)$$

For most of the sequel we omit the index l which distinguishes the different eigenfunctions of ∇^2 on $\mathbf{S}^3$.

In the following subsections we solve equation (11) in some cases of special interest. We then use our results to derive the power spectrum.

3.1.1 Radiation Density Fluctuations in a Radiation Universe

At early stages of expansion and at the end of the collapsing phase, the universe will consist of pure radiation, i.e., all matter will be relativistic. Therefore, this case is important for each hypothetical previous cycle, whether it entered the matter dominated era or it was always radiation dominated. For radiation, where $\omega = c_s^2 = 1/3$, equation (11) reduces to

$$\ddot{D} + \left\{\frac{(l(l+2) - 3)}{3} - \left(\frac{\ddot{a}}{a}\right) - \frac{16}{3}\pi G\rho a^2\right\} D = 0. \quad (12)$$

Inserting the solution of the Friedmann equation for the scale factor of a radiation dominated universe, which is $a(t) = a_{\max}\sin t$ with $a_{\max} = (8\pi G\rho a^4/3)^{1/2}$, we obtain

$$(\sin^2 t)\ddot{D} + \left(\frac{l(l+2)}{3}\sin^2 t - 2\right) D = 0, \qquad t \in [0, \pi]. \quad (13)$$

The solution of this equation is given by

$$D(t) = \sin^2 t\left(\frac{1}{\sin t}\frac{d}{dt}\right)^2 [c_1 \exp\{i\sqrt{a_l}t\} + c_2 \exp\{-i\sqrt{a_l}t\}] \quad (14)$$

with $a_l = l(l+2)/3$ and $l \neq 0$ (see [14]). We are only interested in real solutions for positive integers l, in which case $D(t)$ can be written in the form

$$D(t) = c_1\left[\sqrt{a_l}\cot t\sin(\sqrt{a_l}t) - a_l\cos(\sqrt{a_l}t)\right]$$
$$+ \; c_2\left[\sqrt{a_l}\cot t\cos(\sqrt{a_l}t) - a_l\sin(\sqrt{a_l}t)\right]. \quad (15)$$

This solution is plotted in Fig. 1 for $l = 20$ and $l = 80$. Obviously the amplitude $\cot t$ diverges at the big bang and at the big crunch where $t = 0$ and $t = \pi$ respectively. Since we assume that the fluctuations are created at some time $t_i > 0$ after the big bang, the divergence at $t = 0$ is not a problem. Apart from its oscillation with frequency $\sqrt{a_l} \sim l$, the amplitude of density fluctuations is approximately constant for most of the cycle, but diverges close to the crunch like $D_l(t) \propto l(\pi - t)^{-1}$.

3.1.2 Matter Density Fluctuations in a Radiation Dominated Universe

For dust ($\omega = c_s^2 = 0$) equation (11) reduces to

$$\ddot{D} + \frac{\dot{a}}{a}\dot{D} - 4\pi G\rho_{mat}a^2 D = 0. \tag{16}$$

As long as the universe expands, $\dot{a}$ is positive and hence the second term in this equation acts as a damping term. This term vanishes at maximum expansion and turns into a stimulation when the universe contracts. Therefore we expect the growth of fluctuations to become substantially enhanced during the contraction phase.

Inserting the scale factor $a(t) = a_{max}\sin t$ for the radiation dominated universe, we obtain

$$\sin t\ddot{D} + \cos t\dot{D} - \mu D = 0, \qquad t \in [0, \pi], \tag{17}$$

where $\mu = 4\pi G\rho_{mat}a^3 = (3/2)(a_{max}/a_{eq})$ and $a_{max} = (8\pi G\rho a^4/3)^{1/2}$. With the substitution $x = \sin t$ this equation leads to

$$x(x-1)(x+1)D'' + (2x^2-1)D' + \mu D = 0. \tag{18}$$

This equation is a special case of Heun's differential equation. For $|x| < 1$ one solution

$$D_a(x) = 1 + \sum_{n=1}^{\infty} c_n x^n$$

of (18) is a convergent power series with

$$\begin{aligned} c_1 &= \mu \\ c_{n+1} &= \frac{n(n-1)}{(n+1)^2}c_{n-1} + \frac{\mu}{(n+1)^2}c_n. \end{aligned}$$

A numerical solution of (17) is shown in Fig. 2.

Let us discuss the behavior of these matter density fluctuations more closely during the different epochs of a cycle. At early times, when $t \ll 1$, equation (17) simplifies to

$$t\ddot{D} + \dot{D} - \mu D = 0 \tag{19}$$

which has a solution in terms of Bessel functions:

$$D(t) = c_1 J_0(2i\sqrt{\mu t}) + c_2 Y_0(2i\sqrt{\mu t}) \approx \tilde{c_1} + \tilde{c_2}ln(2\sqrt{\mu t}).$$

Neglecting logarithmic growth, these fluctuations are approximately constant for $t \ll 1$. Fig. 2 shows, that the logarithmic growth for small t is a good approximation up to $t \approx 1/10$ which means that all scales with $l \geq 10$ or so enter the horizon during this era, where we can consider the fluctuations to be approximately constant. This is the well known Mézàros effect: matter fluctuations do not grow in a flat radiation dominated universe. Close to maximum expansion ($t = \pi/2 \pm \epsilon,\ |\epsilon| \ll 1$), equation (17) reduces to

$$\frac{d^2 D}{\epsilon^2} - \epsilon \frac{dD}{\epsilon} - \mu D = 0$$

The solution can be written in terms of the confluent hypergeometric function:

$$D(\epsilon) = \epsilon^{-1/2} e^{\epsilon^2/4} Y\left(\frac{\mu}{2} - \frac{1}{4}, \frac{1}{4}, \frac{-\epsilon^2}{2}\right) , \qquad (20)$$

with

$$Y(k, m, x) = c_1 M_{k,m}(x) + c_2 M_{k,-m}(x)$$

$$M_{k,m}(x) = x^{1/2+m} e^{-x/2} F(1/2 + m - k, 2m + 1, x)$$

$$F(a, b, x) = 1 + \sum_{n=1}^{\infty} \frac{a(a+1)\ldots(a+n-1)x^n}{b(b+1)\ldots(b+n-1)n!}.$$

Near maximum expansion, small $|\epsilon|$, D grows nearly exponentially, like in a non–expanding universe.

Close to the Big Crunch, where $t = \pi - \epsilon$, we again obtain equation (19), replacing t and dots by ϵ and derivatives with respect to ϵ. This reflects the symmetry of the closed universe between (big bang, t) and (big crunch, $-t$) as long as the entropy remains unchanged. Therefore, close to the big crunch D diverges logarithmically:

$$D(t) \propto \ln\left(\frac{1}{2\sqrt{\mu(\pi - t)}}\right).$$

This solution is valid until the particles become relativistic around the de–confinement phase transition where $T \cong 100\,MeV$, from where on we have to consider pure radiation density fluctuations.

3.1.3 Matter Density Fluctuations in a Matter Dominated Universe

In the case of a matter dominated universe, the scale factor is given by $a(t) = (a_{\max}/2)(1 - \cos t)$, where $a_{\max} = 8\pi G\rho a^3/3$ and hence (16) reads

$$(1 - \cos t)\ddot{D} + (\sin t)\dot{D} - 3D = 0, \qquad t \in [0, 2\pi], \qquad (21)$$

with the well known solution (see [15])

$$D(t) = c_1 \left[\frac{5 + \cos t}{1 - \cos t} - \frac{3t \sin t}{(1 - \cos t)^2}\right] + c_2 \left[\frac{\sin t}{(1 - \cos t)^2}\right] \qquad (22)$$

At early times, equation (21) is approximately given by $t^2\ddot{D} + 2t\dot{D} - 6D = 0$, such that $D \propto t^2$ or $D \propto t^{-3}$, for the growing and decaying mode respectively. Close to the collaps, for $t = \pi - \epsilon$, equation (21) reduces again to

$$\epsilon^2 \frac{d^2}{d\epsilon^2} D + 2\epsilon \frac{d}{d\epsilon} D - 6D = 0,$$

but the growing and decaying modes are interchanged. Now the growing mode solution is given by $D = D_0 \epsilon^{-3} = D_0(2\pi - t)^{-3}$. At maximum expansion, the damping term again vanishes and the evolution of the fluctuations around $t = \pi$ is described by exponential growth or decay.

3.1.4 Composite Model

To construct a more realistic model where the scale factor is not only determined by a single matter or radiation background, we now assume a simple composite model, where the energy density of the universe is given by

$$\rho(a) = \frac{\rho_{eq}}{2}\left[\left(\frac{a_{eq}}{a}\right)^3 + \left(\frac{a_{eq}}{a}\right)^4\right].$$

The first expression on the right hand side represents the a^{-3} behavior of the matter density and the second term reflects the a^{-4} behavior of radiation density. The solution of the Friedmann equation (5) in this case is

$$a(t) = \sqrt{\Delta}\sin\left(t - \arcsin\left(\frac{\tilde{a}}{\sqrt{\Delta}}\right)\right) + \tilde{a} = a_{eq}(\alpha \sin t + \alpha^2 \sin^2 \frac{t}{2}), \qquad (23)$$

where $\alpha = a_{eq}/a_0$ with $a_0 = (4\pi G\rho_{eq}/3)^{-1/2}$ and $\tilde{a} = \frac{1}{2}\alpha^2 a_{eq}$, $\Delta = \alpha^2 a_{eq}^2 + \tilde{a}^2$.

Furthermore, we find from (23) that

$$t_{eq} = \arcsin(\Delta^{-1/2}(a_{eq} - \tilde{a})) + \arcsin(\tilde{a}\Delta^{-1/2}),$$

$$t_{max} = \pi/2 + \arcsin(\tilde{a}\Delta^{-1/2}),$$

$$a_{max} \equiv a(t_{max}) = (\Delta^{1/2} + \tilde{a}) = \frac{1}{2}a_{eq}\alpha\left(\alpha + \sqrt{4 + \alpha^2}\right).$$

We can use α as a parameter which determines the duration of the matter dominated epoch in a closed universe containing matter and radiation. For $\alpha \ll 1$, $a_{\max} \approx \alpha a_{eq} \ll a_{eq}$ and the universe never becomes matter dominated. For $\alpha \gg 1$, $a_{\max} \approx \alpha^2 a_{eq} \gg a_{eq}$ and $t_{eq} \ll t_{\max} \approx \pi$; the universe experiences a long matter dominated era.

For radiation density perturbations in this composite model, equation (11) yields

$$a^2(t)\ddot{D}(t) + \left\{a^2(t)\left(\frac{l(l+2) - 3}{3}\right) - a(t)\ddot{a}(t) - 4a_{eq}\tilde{a}\right\} D(t) = 0, \qquad (24)$$

and for matter density perturbations we obtain

$$a(t)\ddot{D}(t) + \dot{a}(t)\dot{D}(t) - 3\tilde{a}D(t) = 0, \tag{25}$$

with $a(t)$ given by (23). We are particularly interested in equation (25). Numerical solutions for the growing mode of a short and long matter dominated phase are shown in Fig. 3.

3.2 The Power Spectrum

3.2.1 The Harrison Zel'dovich initial spectrum

The power spectrum $P(l,t)$ determines the scaling behavior of perturbations at a given time t. It is defined by

$$P(l,t) \equiv |D_l(t)|^2, \tag{26}$$

where $D_l(t)$ is a solution of equation (11). To determine the power spectrum, we have to specify the l–dependence of the initial amplitudes, $D_l(t_{in})$. A preferred such choice, which we also adopt here, is the scale invariant or Harrison–Zel'dovich spectrum [16]. The power spectrum is called Harrison–Zel'dovich if the variance of the mass fluctuation on horizon scales $R_H = \int_0^t dt = t$ is constant, time independent:

$$\langle (\delta M/M)^2_{R_H} \rangle = \text{const.}$$

Here $\langle \cdot \rangle$ denotes the statistical average over many 'realisations' of perturbed universes with identical statistical properties. Since we know only one such universe, we assume that this statistical average can be replaced by a spatial average, a kind of 'ergodic hypothesis'. We want to express $(\delta M)_{R_H}(t)$ as a function of $D_l(t)$. Let us therefore identify the (gauge invariant) density variable $D(x)$ with $(\delta\rho/\rho)(x)$ and let l_H be the value of l corresponding to the horizon size $R_H = \pi/l_H$. Let us denote the spherical harmonics on $\mathbf{S}^3$ by $\mathcal{Y}_\mathbf{k}$, where $\mathbf{k}$ stands for the multi–index (l,j,m) specifying the the spherical harmonics on $\mathbf{S}^3$ (see Appendix A). We then obtain for the mass fluctuation inside a volume of size R_H^3

$$\begin{aligned}(\delta M)_{R_H}(t) &= \int_{V_H} d^3x\, h^{1/2}\delta\rho(\mathbf{x},t) = \rho\int_{V_H} d^3x\, h^{1/2}(\delta\rho/\rho) \\ &= \rho\int_{V_H} d^3x\, h^{1/2}\sum_\mathbf{k}\mathcal{Y}_\mathbf{k}\left(\frac{\delta\rho}{\rho}\right)_\mathbf{k}(t) \approx \rho V_H\sum_{l\le l_H}\sum_{j,m}\mathcal{Y}_\mathbf{k}(\mathbf{x})D_l(t).\end{aligned}$$

For the final approximation, we have assumed that perturbations on scales smaller than π/l_H average to zero due to the integration over V_H, and that perturbations on scales larger than π/l_H are approximately constant in a volume of size $R_H \sim \pi/l_H$, such that integration over V_H just gives rise to the factor V_H (= volume of a three dimensional patch of diameter $2R_H$ on $\mathbf{S}^3$). With $M = \rho V_H$, we then obtain

$$(\delta M/M)^2_{R_H} \approx \sum_{l,l'\le l_H}\ \sum_{(j,m),(j',m')}\mathcal{Y}_\mathbf{k}(\mathbf{x})\mathcal{Y}^*_{\mathbf{k}'}(\mathbf{x})D_l(t)D^*_{l'}(t)$$

and

$$\begin{aligned}\langle(\delta M/M)^2_{R_H}\rangle &\approx \sum_{\mathbf{k},\mathbf{k}'\leq l_H} D_l(t)D^*_{l'}(t)\int_{\mathbf{S}^3} d^3x\, h^{1/2}\mathcal{Y}_{\mathbf{k}}(\mathbf{x})\mathcal{Y}^*_{\mathbf{k}'}(\mathbf{x}) \\ &= \sum_{l\leq l_H}\sum_{j=0}^{l-1}\sum_{m=-j}^{j} P_l(t) = \sum_{l\leq l_H} l^2 P_l(t). \end{aligned} \tag{27}$$

Let $t_l \approx \pi/l$ denote the time when the scale l crosses the horizon, $l_H(t_l) = l$. Since $\sum_{l\leq l_H} l^2 \approx l_H^3$, we see from (27) that we have to demand that

$$P(l, t_l = \pi/l) \propto l^{-3}$$

for $\langle(\delta M/M)^2_{R_H}\rangle$ to be approximately constant, i.e., for a scale invariant spectrum.

The notion of a scale invariant power spectrum can now be used to compare the time evolution of density perturbations on different length scales. We want to investigate, which length scale collapses first. It is the scale at which the variance of the mass perturbation first grows of order unity. At that time, linear perturbation theory breaks down and we expect matter perturbations to form gravitationally bound objects.

3.2.2 The Final Power Spectra

Up to an overall constant, the scale invariant spectrum is determined by the t- and l-dependence of the density perturbations $D_l(t)$ and by the requirement that $D_l(t_l = \pi/l)$ is proportional to $l^{-3/2}$. Then of course $P(l, t_l)$ is proportional to l^{-3} and the variance of the mass perturbation is approximately constant.

Let us first apply this procedure to the case of radiation density fluctuations in the radiation dominated epoch. From solution (15) we find that for a fluctuation which crosses the horizon at times $t \ll \pi$, i.e. at times much smaller than the time of the big crunch, the maximum amplitude of D_l is approximately constant (since the term containing $\cot t$ is small) and therefore c_1 and c_2 must be proportional to $l^{-7/2}$ (since a_l is proportional to l^2) to obtain the required $l^{-3/2}$ behavior of $D_l(t_l)$. Close to the crunch, when $t \to \pi$, the expression containing $\cot t$ diverges as $(\pi - t)^{-1}$. But all scales $l \geq 2$ enter the horizon at times $t_l \leq \pi/2$, and therefore this divergence is only relevant for the mode $l = 1$, which enters the horizon at the big crunch. Of course the scales which enter the horizon already for $t \ll \pi$, also begin to grow as $(\pi - t)^{-1}$, when t approaches π. Disregarding the $l = 1$ mode we thus obtain

$$D_l(t) \propto \begin{cases} l^{-3/2}, & t \ll \pi \\ l^{-3/2}(\pi - t)^{-1}, & t \to \pi, \end{cases} \tag{28}$$

or equivalently

$$P(l, t) \propto \begin{cases} l^{-3}, & t \ll \pi \\ l^{-3}(\pi - t)^{-2}, & t \to \pi \end{cases} \tag{29}$$

for radiation perturbations in the radiation dominated era. At late times, close to the crunch, we can therefore approximate the power spectrum by $P(l,t) \cong c^2 l^{-3}(\pi - t)^{-2}$. This power spectrum obviously takes its maximum for the smallest value of l, and the induced mass fluctuations $l^3 P(l)$ are independent of scale (see Figs. 4A and 4B).

As the next example we consider matter density perturbations in the radiation dominated era. We have found that for $t < 1/10$ they show logarithmic growth which we approximate by a constant. To obtain a scale invariant spectrum we therefore have to require $D_l \propto l^{-3/2}$ and hence again $P(l) \propto l^{-3}$. Only the largest scales which enter the horizon close to or after maximum expansion do not satisfy this proportionality since we can not assume the corresponding density fluctuations to be approximately constant. Since the density perturbations grow with a certain power of t, the slope of the spectrum will decrease towards the largest scales. Matter density fluctuations are a special case of the composite model, when the cycle never reaches the matter dominated phase, $\alpha \ll 1$. The numerically determined power spectrum $P(l)$ and the mass fluctuation $l^3 P(l)$ are shown in curves A and B of Fig. 5.

Now we determine the scale invariant power spectrum for matter density fluctuations in the matter dominated era. For small scales, which enter the horizon early, where $D_l(t) \propto A_l t^2$ scale invariance requires $A_l \propto l^{1/2}$. However when the cycle approaches it's maximum expansion for $t \to \pi$, the damping term is smaller and D_l grows faster, say $D \propto t^\alpha$ with $\alpha > 2$ (around $t_{max} = \pi$ there is actually exponential growth, i.e. α diverges for $l = 1$) and we need $c \propto l^{-3/2+\alpha}$. Towards the crunch, D is proportional to $(2\pi - t)^{-3}$. We finally obtain roughly the following l-dependence of the power spectrum

$$P(l,t) \propto \begin{cases} l(2\pi - t)^{-6}, & l \gg 1, \qquad\qquad 0 \ll 2\pi - t \ll 1 \\ l^{2\alpha-3}(2\pi - t)^{-6}, & l \cong 1, \quad \alpha > 2, \quad 0 \ll 2\pi - t \ll 1 \end{cases}$$

For small l (large scales), the slope of $P(l)$ is bigger than one (in a log-log diagram), where as for large l (small scales), the slope of $P(l)$ is equal to one. This behavior is equivalent to the special case of the composite model with a long matter dominated epoch, $\alpha \gg 1$. Therefore $P(l)$ and $l^3 P(l)$ in the matter dominated universe are very similar to the power spectra shown in Fig. 5 (E) and (F) for $l < 1000$. This figure actually shows a composite model with $\alpha = 1000$. Therefore, scales with $l > 1000$ enter the horizon still in the radiation dominated era and thus do not represent this case. In a pure matter universe, there is no bend in the power spectrum for $l \gg 1$.

Finally we approximate the power spectrum for the realistic composite model. The power spectrum in this case is composed of three parts. At late times, when all scales are already inside the horizon, we obtain ($t_{eq} = \pi / l_{eq}$ denotes the time when $\rho_{mat} = \rho_{rad}$)

$$P(l) \propto \begin{cases} l^{-3}, & l \gg l_{eq} \\ l, & 1 \ll l \ll l_{eq} \\ l^{2\alpha-3}, & l \cong 1, \qquad \alpha > 2. \end{cases} \tag{30}$$

Here, the l^{-3}-dependence is due to fluctuations which enter the horizon already during the radiation dominated epoch like in the flat universe. The maximum of the power spectrum is expected at $l \approx l_{eq}$. In Fig. 5, (A)–(F) some examples for the power spectrum and the corresponding mass fluctuation are plotted. If the cycle has a long matter dominated epoch, the largest scale $l = 1$ enters the horizon soon after maximum expansion of the universe. This is different if the cycle does not reach the matter domination. Then the largest scale enters the horizon very close to the crunch and it will be the scale $l = 2$ which enters the horizon close to maximum expansion.

This leads to the following behavior: if a cycle is purely radiation dominated (i.e. $a < a_{eq}$), the power spectrum takes its maximum for the smallest value of l. If a sequence of cycles approaches and finally enters the matter dominated era, then there will be a 'critical cycle' from which on the maximum of $P(l)$ is no longer the largest scale, $l = 1$ but a scale comparable to $l_{eq} > 1$.

Obviously equation (25) is independent of l and hence the shape of the power spectrum is exclusively determined by the scale invariance condition at horizon crossing and does not change during the subsequent growth of fluctuations. This is not the case, when the particles become relativistic. Then the evolution equation (24) does indeed depend on l and the shape of the power spectrum changes when the cycle approaches the crunch: further local maxima will occur due to the oscillating behavior of solutions of (24), but the global maximum of the power spectrum remains the same. Numerical solutions for this last case are shown in Fig. 6.

In a spatially flat universe containing matter and radiation, the power spectrum for matter density perturbations can be approximated at times $t \gg t_{eq}$ by

$$P(k,t) \cong \frac{C^2 k t^4}{(1 + (k/k_{eq})^2)^2}, \tag{31}$$

which is similar to (30), only that k is continuous and the additional decrease of P for small values of k does not occur (In an open universe the power spectrum even starts to increase for the largest scales). Therefore a measurement of the power spectrum at very large scales (even before a cycle has reached it's maximum expansion) would (in principle) be a way to decide, whether our universe is flat, open or closed. To see the departure of (30) from (31) for large scales, we have plotted both curves together in Fig. 7.

3.3 Interpretation of the Results, Entropy Production in the Previous Cycle

We have thus found that a short time before the big crunch, the mass fluctuation, $\Delta^2(l,t) = l^3 P(l,t)$ is scale independent in a pure radiation universe and decreases towards large scales, $l \leq l_{eq}$ in a matter/radiation universe.

Furthermore, $\Delta^2(l,t)$ diverges at the big crunch. Therefore, at least briefly before the big crunch, linear perturbation theory is no longer applicable. In the pure radiation case, we expect non linear effects to stop gravitational instability and prevent black hole formation at least on small scales. The production of gravitational entropy is thus probably not very significant.

However, if the universe undergoes an intermediate matter dominated period, $\Delta^2(l,t)$ tends to raise towards smaller scales, approaching a very mild, logarithmic growth for $l > l_{eq}$ (see Fig. 5D). We also know from the corresponding flat universe analysis, if fluctuations grow non–linear before, due to contraction, the universe becomes radiation dominated again, non–linear gravity and the log–raise towards small scales will lead to the collaps of small scales and probably to the formation of small black holes.

If we want to prevent excessive black hole and entropy formation in the cycle previous to the present one, we thus have to require that perturbations never get strongly nonlinear. This yields a limit for the radiation entropy in the previous cycle. To illustrate this, let us assume that in the present cycle perturbations get non–linear, $\Delta^2(l,t) \approx 1$ around a redshift of $z \sim 10$, $T \sim 30K$. For this not to happen in the previous cycle, we have thus to require $T_{\min} > 30K$ or $S = (\lambda_2/T_{\min})^3 < 10^{84}$.

The radiation entropy of the previous cycle thus has to be at least a factor of 10^3 times smaller than the present entropy. We therefore require that most of the radiation entropy of the universe at present, $S \geq 10^{87}$, was produced in the form of gravitational entropy from small fluctuations during the previous cycle. Unfortunately, we do not have a quantitative description for the entropy of the gravitational field (except in the case of black holes), but it is certainly related to the clumpiness of the matter which is determined by the Weyl part of the curvature [3].

We now postulate, that during the quantum gravity epoch of big crunch/big bang passage the entropy of the gravitational field is completely transformed into radiation entropy and the new cycle starts out from a state with vanishing gravitational entropy, a homogeneous and isotropic Friedmann Lemaître universe. At first this postulate might seem somewhat ad hoc, but it is actually just what happens if a black hole evaporates due to Hawking radiation.

It is thus feasible that most of the entropy production in the universe is actually due to mild gravitational clustering in the previous cycle and not due to local non–thermal processes.

4 Conclusions

We have revisited the oscillating universe and shown how it can yield a coherent solution to the flatness or entropy and the horizon problems of standard cosmology. We have analized linear gravitational perturbations in a closed universe consisting of matter and radiation. We can set an upper limit on the radiation entropy of the previous cycle which is at least a factor 10^3 below the entropy of the present universe. We thus postulate that most of the radiation entropy in the present cycle was produced as gravitational entropy by linear or mildly non–linear gravitational clustering in the previous cycle. During the quantum gravity big crunch/ big bang era, this gravitational entropy must then be transformed into radiation entropy.

Due to the lack of of a theory of quantum gravity, we have no precise idea how this is accomplished. Nevertheless, this is exactly what happens if black holes evaporate!

The reader may now object that we postulate the emergence of a Friedmann Lemaître universe out of the quantum gravity era, whereas homogeneity and isotropy is naturally obtained in some inflationary models, e.g. chaotic inflation. However also in chaotic inflation, where homogeneity and isotropy is achieved by blowing up small scales, one has to require a cut–off of fluctuations at some very small scale, typically around Planck scale.

We consider it to some extend a matter of taste which of the two requirements for quantum gravity is more 'restrictive'; that it leads to a cut–off of fluctuations around the Planck scale or that it leads to the transformation of gravitational entropy into radiation entropy in very high curvature regions. Nevertheless, it is a weakness of our model, that we cannot propose a clear picture of how this transformation might take place. We plan to address this problem in the future.

In our approach the creation of initial density fluctuations is not addressed. They might be created in (or left over from) the Planck era or they might build up during a phase transition in the early universe (topological defects) or by any other scale invariant process, like the self ordering of a global scalar field on Hubble scale.

Clearly, the easiest way to falsify this model would be to measure $\Omega < 1$. On the other hand, if $\Omega \equiv 1$ we will never be able to decide whether $\Omega = 1 + \epsilon$ or $\Omega = 1 - \epsilon$, and other means to distinguish this scenario from, e.g., inflationary models have to be developed.

Appendix

A Scalar Harmonic Functions

In the closed universe scalar quantities like D can be expanded in a complete set of scalar harmonic functions $\mathcal{Y}_{\mathbf{k}}(\mathbf{x}) = \mathcal{Y}_{\mathbf{k}}(\chi, \theta, \phi)$ on the three sphere $\mathbf{S}^3$:

$$D(x) = \sum_{\mathbf{k}} \mathcal{Y}_{\mathbf{k}}(\chi, \theta, \phi) D_{\mathbf{k}}(t), \qquad (32)$$

where $\mathbf{k} = (l, j, m)$, $l = 0, 1, 2, 3, \ldots$, $j = 0, 1, \ldots, l - 1$, $m = -j, -j + 1, \ldots, j$. The variables $\chi \in [0, \pi]$, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$ denote the angles describing the position on the three sphere. The functions $\mathcal{Y}_{\mathbf{k}}$ satisfy the Laplace-Beltrami equation with eigenvalue $-k^2$:

$$(\Delta + k^2)\mathcal{Y}_{\mathbf{k}} = 0.$$

Here $\Delta \equiv \nabla^j \nabla_j$ denotes the three-dimensional Laplacian on $\mathbf{S}^3$, $k^2 = l(l+2)K$ and for our case of interest $K > 0$ (In most of the sequel we set $K = 1$). The harmonic functions $\mathcal{Y}_{\mathbf{k}}(\mathbf{x})$ are given by

$$\mathcal{Y}_{\mathbf{k}}(\mathbf{x}) = \Pi^{(+)}_{\beta j}(\chi) Y_{jm}(\theta, \phi), \qquad \beta^2 = k^2 + K = (l+1)^2 K,$$

where $Y_{jm}(\theta, \phi)$ are the usual spherical harmonics on $\mathbf{S}^2$ and the $\Pi^{(+)}_{\beta j}$ can be expressed in terms of generating functions

$$\Pi^{(+)}_{\beta j}(\chi) = i^j \frac{\sin^j \chi}{(M^j_\beta)^{1/2}} \left(\frac{d}{d \cos \chi} \right)^{j+1} \cos(\beta \chi),$$

where M^j_β is the normalization factor

$$M^j_\beta = (\pi/2) \prod_{n=0}^{l-1} (l^2 - n^2).$$

The normalization of the functions $\mathcal{Y}_{\mathbf{k}}(\mathbf{x})$ is as usual

$$\int_{\mathbf{S}^3} d^3x \, h^{1/2}(\mathbf{x}) \mathcal{Y}_{\mathbf{k}}(\mathbf{x}) \mathcal{Y}^*_{\mathbf{k}'}(\mathbf{x}) = \delta_{\mathbf{k},\mathbf{k}'},$$

where $h(\mathbf{x})$ is the determinant of the 3-metric of constant curvature $K = 1$ and $\delta_{\mathbf{k},\mathbf{k}'}$ is the Kronecker delta

$$\delta_{\mathbf{k},\mathbf{k}'} = \begin{cases} 1, & \text{if} \quad \mathbf{k} \equiv (l, j, m) = \mathbf{k}' \equiv (l', j', m') \\ 0, & \text{else.} \end{cases}$$

Furthermore we choose the phases of $\mathcal{Y}_{\mathbf{k}}$ such that $\mathcal{Y}^*_{\mathbf{k}}(\mathbf{x}) = \mathcal{Y}_{-\mathbf{k}}(\mathbf{x})$, with $-\mathbf{k} \equiv (l, j, -m)$. (See e.g. [17] or [18] for further details).

B The Passage between two Cycles

In this Appendix we want to show briefly, how the transition from a Big Crunch to a subsequent Big Bang can be described by an effective model. The main idea is the appearance of a signature change in the metric from Lorentzian to Euclidian and back. By this mechanism, the singular behavior of spacetime at $a = 0$ disappears and the topology of the transition region is that of $\mathbf{S}^4$.

In analogy to the change of signature idea of Hartle & Hawking [19] in quantum cosmology, Ellis [11] and Ellis et. al. [12] have shown that the classical Einstein field equations allow a change of signature when the metric is allowed to possess a mild singularity. The classical case leads to interesting possibilities for the description of an oscillating universe.

The signature change is implemented into the metric by introduction of a lapse function $n(\tau)$:

$$ds^2 \quad = \quad -n(\tau)d\tau^2 + a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]. \tag{33}$$

Here τ denotes cosmic time: $d\tau = a\, dt$. For the discontinuous choice of the lapse function $n(\tau) = \epsilon$ with $\epsilon = \pm 1$, there exists a surface of change Σ, where the metric changes its signature. From (33) and Einstein's equations one derives the Friedmann and Raychaudhuri equations for the scale factor $a(\tau)$, which then hold in the regions V_+, where $\epsilon = +1$ and in V_-, where $\epsilon = -1$, but not on the surface of signature change Σ, since there the metric tensor is not invertible. By choosing suitable (physically motivated) jump conditions on Σ, one can find solutions for the scale factor which pass continuously through the surface of signature change [11, 12].

For the simple case of a scalar field $\phi \in \mathbf{R}$ with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi),$$

such that $\dot{\phi} = 0$ (the no rolling case), Ellis et. al. have shown that the scale factor behaves for $k = 1$ like

$$a(t) = \begin{cases} H^{-1}\cos(H\tau), & -\pi/(2H) \le \tau \le 0 \\ H^{-1}\cosh(H\tau), & \tau \ge 0\ . \end{cases} \tag{34}$$

The corresponding space-time has no boundary and is geodesically complete (i.e. it has no singularity and the geodesics can be continued smoothly through the surface of signature change). Only the length of the tangent vector jumps for photons and spacelike geodesics at the surface of signature change Σ. Obviously the scale factor given by (34) inflates for $\tau > 0$. We do not have an equivalent simple example with the same nice features which does not inflate. However there are other (rolling) solutions to get a successful exit from inflation, but these solutions do not have the 'no-boundary' property of the above mentioned case. For a detailed discussion see [11] and [12]. Our main point is, that there is a possibility to continue the evolution of the universe smoothly through the crunch which might serve as an effective theory for the passage between two subsequent cycles.

B.1 Geometric Representation

It is interesting to note that only geodesics describing massive particles at rest pass through the point where $a = 0$, whereas a particle with velocity $v_\Sigma \neq 0$ on Σ enters the Euclidian regime with an angle α as shown in Fig. 8 coordinates shown are t and r. The angles θ and ϕ are suppressed, since we assume them to be constant for the indicated geodesic). The coordinates of the plane with angle α in the $y - z$ plane, are

$$(x, y, z) = (x, -z\tan\alpha, z).$$

With polar coordinates chosen as

$$x = \cos\theta\cos\varphi, \quad y = \cos\theta\sin\varphi, \quad z = -\sin\theta,$$

where $\varphi \in [0, \pi]$ and $\theta \in [0, \pi/2 - \alpha]$, one finds

$$\cos\theta\sin\varphi = \sin\theta\tan\alpha.$$

Hence a geodesic on $\mathbf{S}^2$ through $\varphi = \theta = 0$ is given by

$$\varphi(\theta, \alpha) = \arcsin[\tan\theta \cdot \tan\alpha].$$

Here $\theta = H\tau$ is the time coordinate. The velocity v_Σ on Σ then is

$$v_\Sigma \equiv \frac{d\varphi}{d\theta}|_{\theta=0} = \tan\alpha,$$

indicating that a point particle which is not at rest, does not reach the point where $a = 0$. A photon with $v_\Sigma = 1$ enters with the angle $\alpha = \arctan 1$ which is $\pi/4$. Hence a photon is not represented by the boundary at $\alpha = \pi/2$, but by a line inside the Euclidian regime.

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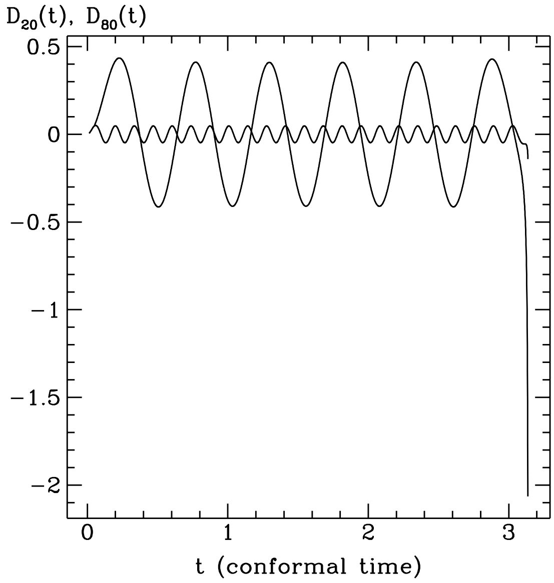

Figure 1: The time evolution of $D_l(t)$ for radiation density perturbations in a radiation dominated universe (in arbitrary units). The amplitude $D_l(t)$ is shown for the scales $l = 20$ and $l = 80$. Very close to the big crunch, the divergence due to $\cot t$ takes over.

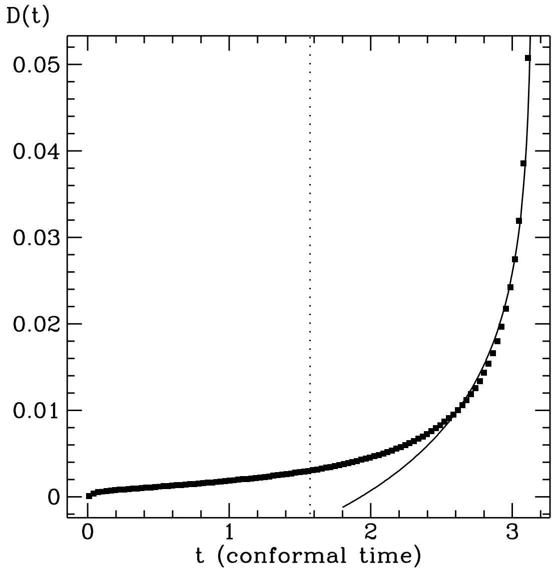

Figure 2: The time evolution of $D(t)$ for matter density perturbations in a radiation dominated universe (in arbitrary units). For comparison, a logarithmically divergent fit is shown close to the collaps (solid line). The vertical dotted line indicates t_{max}, the time when the cycle reaches its maximum expansion.

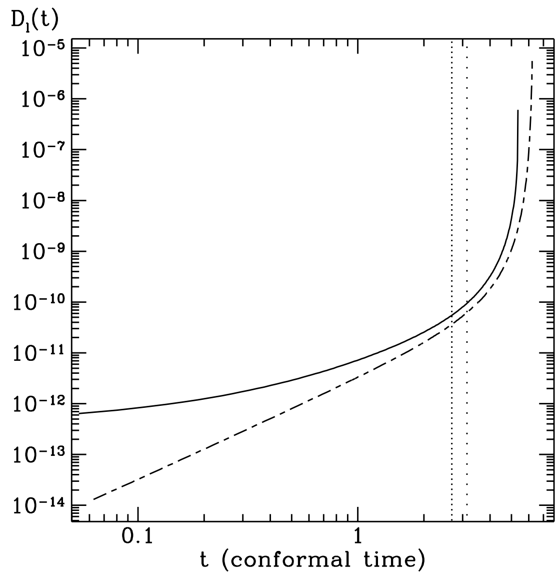

Figure 3: The amplitude $D(t)$ of matter density perturbations in the composite model (in arbitrary units). The solid line shows $D(t)$ for a cycle with a short matter dominated phase ($\alpha = 4$). The dashed line shows $D(t)$ for a cycle with a long matter dominated epoch ($\alpha = 1000$, $a_{max} \gg a_{eq}$). The left and right vertical dotted lines indicate the time of maximum expansion of the cycle for $\alpha = 4$ and $\alpha = 1000$ respectively.

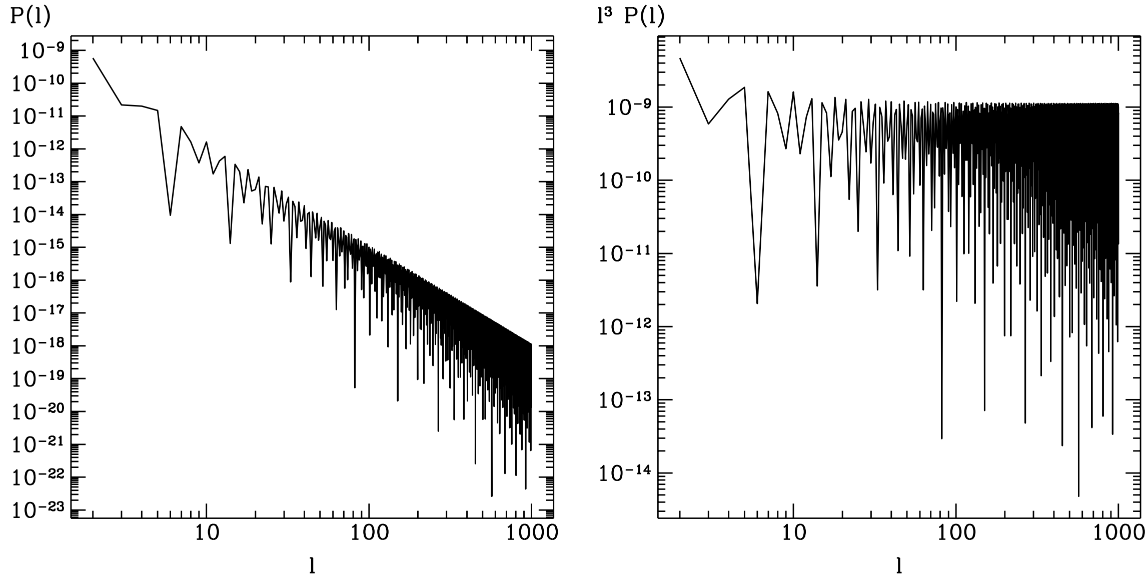

Figure 4: (**A**): The power spectrum $P(l)$ as a function of l for radiation density perturbations in the radiation dominated universe (in arbitrary units). $P(l)$ is given at a time, when all scales $l \geq 2$ are inside the horizon.
(**B**): The corresponding mass fluctuation $l^3 P(l)$ as a function of l.

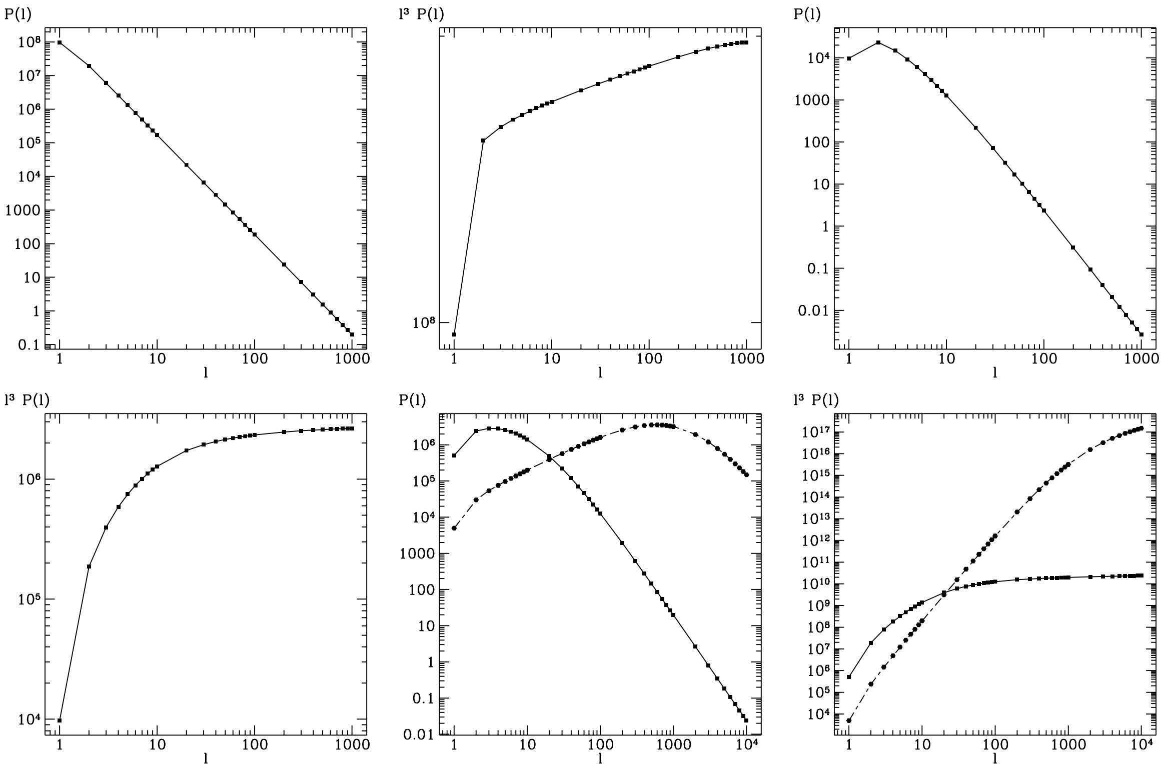

Figure 5: The power spectrum $P(l)$ and the induced mass fluctuation $l^3P(l)$ as a function of l for matter density perturbations in the composite model (in arbitrary units) The lines connecting the points $l \in \mathbf{N}$ are shown for clarity.
(**A**) & (**B**): A purely radiation dominated cycle, $\alpha \ll 1$, $a_{max} \ll a_{eq}$.
(**C**) & (**D**): A cycle which just reaches the matter dominated epoch, $\alpha = 1$, $a_{max} = a_{eq}$.
(**E**) & (**F**): A cycle including a short matter dominated epoch with $\alpha = 4$ (square points, maximum of $P(l)$ in (**E**) at $l \sim 4$) and a cycle including a long matter dominated epoch ($a_{max} \gg a_{eq}$) with $\alpha = 1000$ (hexagonal points, maximum of $P(l)$ in (**E**) at $l \sim 1000$).

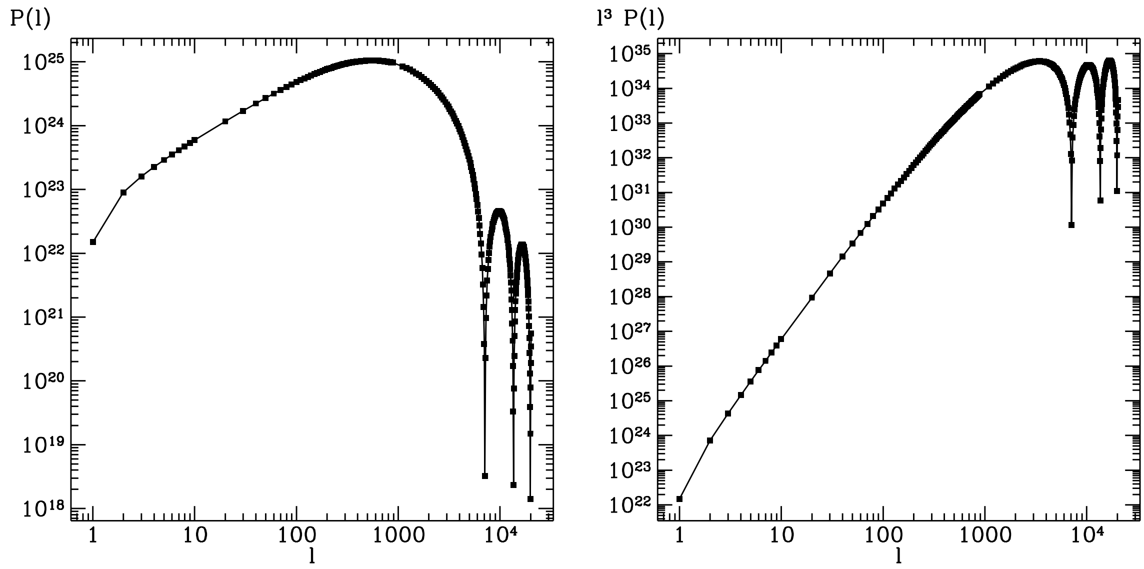

Figure 6: (**A**): The power spectrum $P(l)$ in the composite model as a function of l, when matter becomes relativistic close to the crunch (arbitrary units). The cycle includes a long matter dominated epoch $\alpha = 1000$ (The solid line simply connects the evaluated points).
(**B**): The induced mass fluctuation $l^3 P(l)$ for the same cycle as in (**A**).

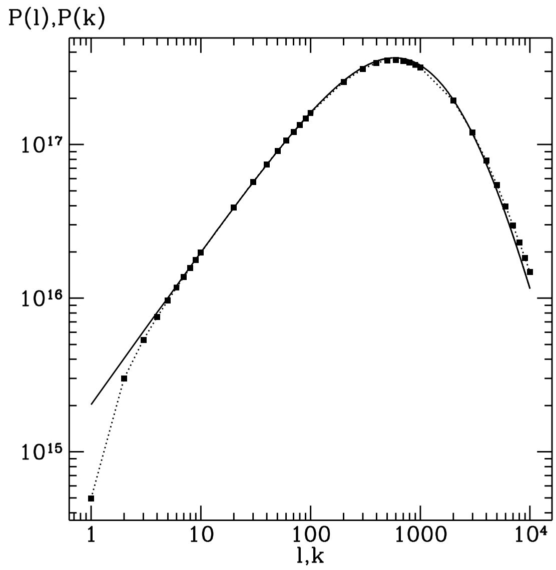

Figure 7: The discrete power spectrum $P(l)$ for the composite model with $\alpha = 1000$ (square points connected by dots) in comparison with the continuous flat space analogon $P(k)$ (solid line).