STRUCTURE FORMATION IN THE UNIVERSE FROM TEXTURE INDUCED FLUCTUATIONS

Ruth Durrer and Zhi–Hong Zhou

Universität Zürich, Institut für Theoretische Physik,
Winterthurerstrasse 190,
CH-8057 Zürich, Switzerland

Abstract

We discuss structure formation with topological defects. First we present a partially new, local and gauge invariant system of perturbation equations to treat microwave background and dark matter fluctuations induced by topological defects or any other type of seeds. We show that this system is well suited for numerical analysis of structure formation by applying it to the texture scenario. Our numerical results cover a larger dynamical range than previous investigations and are complementary to them since we use substantially different methods.

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Despite great effort and considerable progress, the problem of structure formation in the universe remains basically unsolved. Observations show that the fluctuation spectrum on large scales observed by COBE should be not very far from scale invariant [1]. This has been considered as great success for inflationary models which predict a scale invariant fluctuation spectrum. In this letter we consider an alternative class of models which also yield a scale invariant spectrum of Cosmic Microwave Background (CMB) fluctuations: Models where perturbations are seeded by global topological defects which can form during symmetry breaking phase transitions in the early universe [2]. To be specific, we consider texture, π_3–defects which lead to event singularities in four dimensional spacetime [3]. A common feature of global topological defects is the behavior of the energy density of the scalar field which scales like $\rho_T \propto 1/(at)^2$ and thus always represents the same fraction of the total energy density of the universe.

$$\rho_T/\rho \sim 8\pi G\eta^2 \equiv 2\epsilon \ , \tag{1}$$

where η determines the symmetry breaking scale. The background spacetime is a Friedmann–Lemaître universe with $\Omega = 1$. We choose conformal coordinates such that

$$ds^2 = a^2(-dt^2 + \delta_{ij}dx^i dx^j) \ .$$

Numerical analysis of CMB fluctuations from topological defects on large scales has been performed in [4, 5]; a spherically symmetric approximation is discussed in [6]. Results for intermediate scales are presented in [7]. All these investigations (except [6]) use linear cosmological perturbation theory in synchronous gauge and take into account only scalar perturbations. Here we derive a fully gauge invariant and local system of perturbation equations. The (non–local) split into scalar, vector and tensor modes on hypersurfaces of constant time is not performed. We solve the equations numerically in a cold dark matter (CDM) universe with global texture. In this letter, we present some main results, detailed derivations of the equations and a description of our numerical methods will be given in a longer paper [8]. Since there are no spurious gauge modes in our initial conditions, there is no danger that these may grow in time and much of the difficulties to choose correct initial conditions (see e.g. [5]) are removed.

We calculate the CMB anisotropies on angular scales which are larger than the angle subtended by the horizon scale at decoupling of matter and radiation, $\theta > \theta_d$. For $\Omega = 1$ and $z_d \approx 1000$

$$\theta_d = 1/\sqrt{z_d + 1} \approx 0.03 \approx 2^o \ . \tag{2}$$

It is therefore sufficient to study the generation and evolution of microwave background fluctuations after recombination. During this period, photons stream freely, only influenced by cosmic gravitational redshift and by perturbations in the gravitational field (if the medium is not reionized). The photon distribution function which lives on a seven dimensional relativistic phase space $P_0\mathcal{M} = \{(x,p) \in T\mathcal{M} | g(x)(p,p) = 0\}$, obeys Liouville's equation

$$X_g(f) = 0 \ . \tag{3}$$

In a tetrad basis e_μ, X_g is given by (see e.g. [9])

$$X_g = (p^\mu e_\mu + \omega^i_{\ \mu}(p)p^\mu \frac{\partial}{\partial p^i}) \ , \tag{4}$$

where $\omega^\nu_{\ \mu}$ are the connection 1–forms of $(\mathcal{M}, g)$ in the basis e^μ.

The metric of a perturbed Friedmann universe with $\Omega = 1$ is given by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ with

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}) = a^2\tilde{g}_{\mu\nu} \ , \tag{5}$$

where $(\eta_{\mu\nu}) = diag(-,+,+,+)$ is the flat Minkowski metric and $|h_{\mu\nu}| \ll 1$ is a small perturbation. We now use the fact that the motion of photons is conformally invariant. Taking into account the different affine parameters, (3) is equivalent to

$$(X_{\tilde{g}}f)(x, ap) = 0 \ . \tag{6}$$

If $\bar{e}^\mu$ is a tetrad in Minkowski space, $e_\mu = \bar{e}_\mu + (1/2)h^\nu_\mu \bar{e}_\nu$ is a tetrad w.r.t the perturbed geometry $\tilde{g}$. We can thus define the perturbation of the distribution function F by

$$f(x, p^\mu e_\mu) = \bar{f}(x, p^\mu \bar{e}_\mu) + F(x, p^\mu \bar{e}_\mu) \ . \tag{7}$$

Furthermore, we set $p^i = p\gamma^i$, with $p^2 = \sum_{i=1}^3 (p^i)^2$ and $v = ap$. Liouville's equation for f then yields a perturbation equation for F. We choose the natural tetrad $e_\mu = \partial_\mu + (1/2)h^\nu_\mu \partial_\nu$. Using (3),(4) and (6) we obtain

$$(\partial_t + \gamma^i\partial_i)F = -[\dot{H}_L + (A_{,i} + \frac{1}{2}\dot{B}_i)\gamma^i + (\dot{H}_{ij} - B_{i,j})\gamma^i\gamma^j]v\frac{d\bar{f}}{dv} \ , \tag{8}$$

where we parametrize h by

$$(h_{\mu\nu}) = \begin{pmatrix} 2A & B_i \\ B_i & 2H_L\delta_{ij} + 2H_{ij} \end{pmatrix} , \tag{9}$$

with $H^i_i = 0$.

Let $\iota = a^{-4}\int f v^3 dv$ be the energy integrated brightness and

$$m = (1/4)\frac{4\pi}{\rho_r a^4}\int F v^3 dv \ .$$

$4m$ is the fractional perturbation of ι. Setting $\iota = \bar{\iota}(T(\gamma, x))$, one finds that m corresponds to the fractional perturbation in the temperature,

$$T(\gamma, x) = \bar{T}(1 + m(\gamma, x)) \ . \tag{10}$$

(A more explicit derivation of (10) is given in [10]). Integrating (8) over v with weight v^3dv, we obtain

$$\partial_t m + \gamma^i\partial_i m = \dot{H}_L + (A_{,i} + \frac{1}{2}\dot{B}_i)\gamma^i + (\dot{H}_{ij} - B_{i,j})\gamma^i\gamma^j \ . \tag{11}$$

It is well known that photons only couple to the Weyl part of the curvature (null geodesics are conformally invariant). The r.h.s. of (11) is given by first derivatives of the metric only which could at most represent integrals of the Weyl tensor. To obtain a local, non–integral equation, we thus rewrite (11) in terms of $\nabla^2 m$. In fact, defining

$$\chi = \nabla^2 m - (\nabla^2 H_L - \frac{1}{2} H_{,ij}^{ij}) - \frac{1}{2}\nabla^2 B_i \gamma^i ,$$

(11) yields for χ the equation of motion

$$(\partial_t + \gamma^i \partial_i)\chi = -3\gamma^i \partial^j E_{ij} - \gamma^k \gamma^j \epsilon_{kli} \partial_l B_{ij} \equiv S_T(t, \boldsymbol{x}, \boldsymbol{\gamma}) , \tag{12}$$

where ϵ_{kli} is the totally antisymmetric tensor in three dimensions, E_{ij} and B_{ij} are the electric and magnetic part of the Weyl tensor. The spatial indices in this equation are raises and lowered with δ_{ij} and therefore the index position is irrelevant. Double indices are summed over irrespective of their position. Eqn. (12) is the central analytical result of this letter.

The divergence of the electric part of the Weyl tensor does not contain tensor perturbations. On the other hand, scalar perturbations do not induce a magnetic gravitational field. The second contribution to the source term in (12) represents a combination of vector and tensor perturbations. If vector perturbations are negligible, the two terms on the r.h.s. of (12) represent a split into scalar and tensor perturbations which is local. We find that the vector and tensor fluctuations represented by B contribute approximately 25 % to the total microwave background anisotropies (see Fig. 1).

In terms of metric perturbations, the electric and magnetic part of the Weyl tensor are given by (see, e.g. [11])

$$E_{ij} = \frac{1}{2}[\triangle_{ij}(A - H_L) - \dot{\sigma}_{ij} - (\nabla^2 H_{ij} + \frac{2}{3} H_{lm}^{,lm} \delta_{ij}) - H_{il}^{,l}{}_{,j} - H_{jl}^{,l}{}_{,i}] \tag{13}$$

$$B_{ij} = \frac{1}{2}(\epsilon_{ilm}\sigma_{jm,l} + \epsilon_{jlm}\sigma_{im,l}) , \tag{14}$$

$$\text{with} \quad \sigma_{ij} = \frac{1}{2}(B_{i,j} + B_{j,i}) - \frac{1}{3}\delta_{ij} B_l'^{l} - \dot{H}_{ij} \quad \text{and} \quad \triangle_{ij} = \partial_i\partial_j - (1/3)\delta_{ij}\nabla^2 .$$

Since the Weyl tensor of Friedmann–Lemaître universes vanishes, the r.h.s. of (12) is manifestly gauge invariant (this is the so called Stewart–Walker lemma [14]). Therefore also the variable χ is gauge invariant.

The general solution to (12) is given by

$$\chi(t, \boldsymbol{x}, \boldsymbol{\gamma}) = \int_{t_i}^{t} S_T(t', \boldsymbol{x} + (t' - t)\boldsymbol{\gamma}, \boldsymbol{\gamma})dt' \; + \; \chi(t_i, \boldsymbol{x} + (t_i - t)\boldsymbol{\gamma}, \boldsymbol{\gamma}) , \tag{15}$$

where S_T is the source term given on the rhs of (12).

The electric and magnetic part of the Weyl tensor are determined by the perturbations in the energy momentum tensor via Einstein's equations. We assume that the source for the geometric perturbations is given by the scalar field and dark matter. The contributions from radiation may be neglected. Furthermore, vector perturbations of dark matter (which decay quickly) are neglected. The divergence of E_{ij} is then determined by (see, [11] or [12])

$$\partial^j E_{ij} = -8\pi G\rho_{DM}D_i - 8\pi G(\partial_i\delta T_{00} + 3(\frac{\dot{a}}{a})\delta T_{0i}) + 12\pi G\partial^j\tau_{ij} \ , \qquad (16)$$

where

$$\tau_{ij} \equiv T_{ij} - (a^2/3)\delta_{ij}T_l^l = \tau_{ij}^{(texture)} = \phi,_i \phi,_j - (1/3)\delta_{ij}(\nabla\phi)^2 \ ,$$

$$\delta T_{0j} = \delta T_{0j}^{(texture)} = \dot{\phi}\phi,_j \ ,$$

$$\delta T_{00} = \delta T_{00}^{(texture)} = \frac{1}{2}((\dot{\phi})^2 + (\nabla\phi)^2) \ ,$$

and D_j is a gauge invariant perturbation variable for the density gradient (see [11, 12, 8]). For scalar perturbations $D_j = \partial_j D$. The evolution of the dark matter density perturbation is governed by

$$\ddot{D} + (\frac{\dot{a}}{a})\dot{D} - 4\pi G a^2\rho_{DM}D = 8\pi G\dot{\phi}^2 \ . \qquad (17)$$

The equation for B_{ij} is more involved. A somewhat cumbersome derivation [8] yields

$$\ddot{B}_{ij} + 3(\frac{\dot{a}}{a})\dot{B}_{ij} - \nabla^2 B_{ij} = 8\pi G\mathcal{S}_{ij}^{(B)} \ , \qquad (18)$$

$$\text{with} \quad \mathcal{S}_{ij}^{(B)} = \epsilon_{lm(i}\delta T_{0l},_{j)m} - (\dot{a}/a)\epsilon_{lm(i}\tau_{j)l},_m \ .$$

Here $(i...j)$ denotes symmetrization in the indices i and j.

These equations are complemented with the evolution equation of the scalar field,

$$\ddot{\phi} + 2(\dot{a}/a)\dot{\phi} - \nabla^2\phi = a^2\lambda\phi(\phi^2 - \eta^2) \ . \qquad (19)$$

We have solved the closed hyperbolic system (12, 16, 17, 18, and 19) numerically on a 192^3 grid for different initial conditions on a NEC–SX3. The numerical methods employed and the different tests of our programs are described in [8]. Here we just want to present some main results.

On subhorizon scales the gauge dependent variable $\delta\rho/\rho$ D do not differ substantially, and we can interpret D by

$$D \approx (\delta\rho/\bar{\rho}) = \delta \ .$$

Furthermore, we note that $m = \delta T/T$ differs from χ only by a monopole and a dipole term:

$$\chi = \nabla^2(\delta T/\bar{T}) + \text{ monopole term } + \text{ dipole term} \equiv \nabla^2(\Delta) \ .$$

Since we are only interested in spherical harmonic amplitudes a_{lm} with $l \geq 2$, this difference is irrelevant for CMB fluctuations. (For a fixed observer, the monopole term cannot be distinguished from the background, and a dipole term cannot be separated from peculiar motion.). Using fast Fourier transforms we calculate the spectrum $P(k) = |\delta(k)|^2$ and Δ, which we then expand in spherical harmonics

$$\Delta(t_0, \boldsymbol{x}, \boldsymbol{\gamma}) = \sum_{lm} a_{lm}(x) Y_{lm}(\boldsymbol{\gamma}) \ . \tag{20}$$

As usual, we assume that the average over N_x different observer positions coincides with the ensemble average and determine

$$c_l = \frac{1}{(2l+1)N_x} \sum_{m,x} |a_{lm}(\boldsymbol{x})|^2 \ , \quad l \geq 2 \ . \tag{21}$$

We have performed 10 simulations on a 192^3 grid with about 100 different observer positions for each simulation. The average harmonic amplitudes with 1σ variance are shown in Fig. 2. The low order multipoles depend strongly on the random initial conditions (cosmic variance), like in the spherically symmetric simulation [6]. From Fig. 2 it is clear that the texture scenario is compatible with a scale invariant spectrum. The main difference of the currently favored inflationary scenarios lies in the distribution of fluctuations which is non Gaussian in models with topological defects. The quadrupole amplitude is given by

$$Q = (0.53 \pm 0.16)\epsilon \ .$$

To reproduce the COBE amplitude $Q_{COBE} = (0.6 \pm 0.1)10^{-5}$ [1], we have to normalize the spectrum by choosing the phase transition scale η according to

$$\epsilon = 4\pi G \eta^2 = (1.1 \pm 0.5)10^{-5} \ . \tag{22}$$

This value is comparable with the value of ϵ obtained in [4, 5].

The power spectrum of dark matter density fluctuations is shown in Fig. 3. To be compatible with observations [15], a somewhat high bias factor of $b \approx 3 \pm 1.5$ is required. (The bias factor takes into account that the observed clustering of light does not necessarily coincide with the clustering of dark matter.) Observations and simulations of nonlinear clustering of dark matter and baryons [16] suggest a bias factor $b \sim 2$.

In this letter we have presented a closed system of cosmological perturbation equations which is not plagued by gauge modes and which is well suited for numerical analysis. Up to some details, our numerical results are consistent with previous investigations [4, 5, 6], indicating that the texture scenario normalized to the COBE DMR experiment yields somewhat too much power on small scales.

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Figures

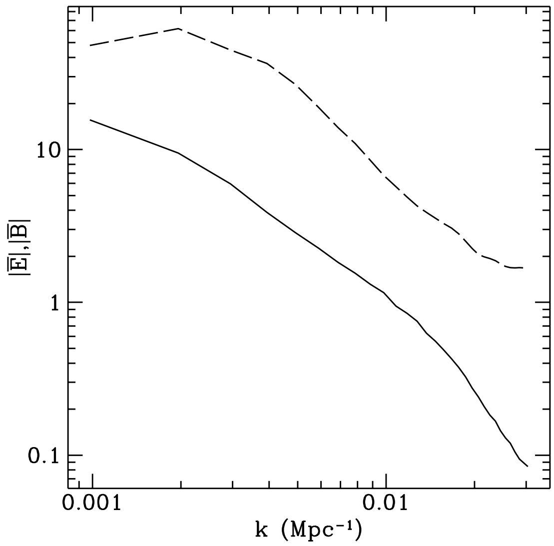

Figure 1: The amplitude of the electric and magnetic source terms to the photon equation if motion are shown as a function of wavenumber k in arbitrary scale. On very large scales the magnetic part contributes about 1/4 decaying to roughly 1/10 on small scales ($\bar{E} = \frac{1}{3}\sum_i(\partial^j E_{ij})^2$, $\bar{B} = \frac{1}{6}\sum_{ij}(\epsilon_{jlk}\partial^l B_{ki})^2$). ($\bar{B}$ is shown as solid line.)

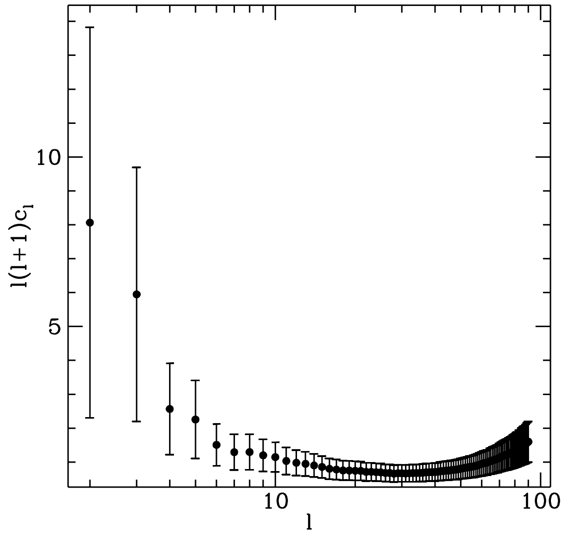

Figure 2: The harmonic amplitudes $l(l+1)c_l$ are shown with 1σ error bars. The slight rise at large l is due to the finite size of the texture core. No smoothing is applied.

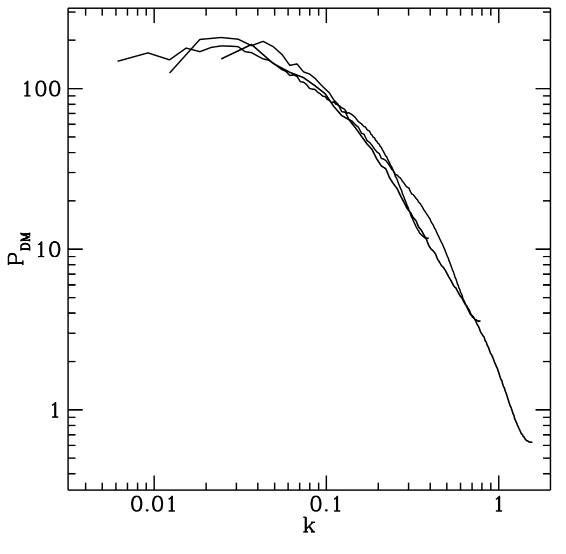

Figure 3: The power spectrum of the CDM fluctuations induced by texture. To enhance the dynamic range, three simulations with different physical grid size have been patched together. The vertical scale is arbitrary.