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Effects of biasing on the matter power spectrum at large scales

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Abstract: In this paper we study the effect of biasing on the power spectrum at large scales. We show that even though non-linear biasing does introduce a white noise contribution on large scales, the $P(k) \propto k^n$ behavior of the matter power spectrum on large scales, may still be visible and above the white noise for about one decade. We show, that the Kaiser biasing scheme which leads to linear bias of the correlation function on *large* scales generates a linear bias of the power spectrum on *small* scales (large wave number). We also discuss the effect of biasing on the baryon acoustic oscillations.

Keywords: Cosmological large scale structure, biasing, baryon acoustic oscillations.

Contents

1. Introduction

One of most promising future observations for cosmology is the precise determination of the galaxy power spectrum. So far, the emphasis has been on the anisotropies and polarization of the cosmic microwave background (CMB) but in the future we also want to determine the matter distribution of the Universe with much better precision. The advantage of the matter distribution if compared to the CMB is that while the latter represents only a two-dimensional data set, it has been emitted from the surface of last scattering, the former is three-dimensional and therefore contains, in principle much more information. The disadvantage is that we can only observe galaxies and it is not clear how the observed galaxy correlation function is related to the underlying matter correlation function which we calculate using cosmological perturbation theory (see e.g. [1]) and numerical N-body simulations (see e.g. [2]).

This problem goes under the name of 'biasing' and has been studied in many works. Among the first influential papers on the topic are [3, 4, 5]. Here we want to address two main issues of biasing. First, we want to illustrate the fact that even though the galaxy correlation function may differ from the matter correlation function by more than a simple multiplicative constant (non-linear biasing) only on small scales, this can significantly modify the galaxy power spectrum also on large scales.

Secondly we want to investigate the effect that biasing can have on the baryon acoustic oscillations (BAO's). This are the remains of the oscillations in the baryon-photon plasma generated prior to recombination which then contribute to the matter power spectrum. These acoustic peaks have been measured in detail in the CMB anisotropy spectrum [8], and there is evidence of their presence also in the galaxy power spectrum [9]. There the signal is not at very high significance, about 2 sigma, but substantial progress is expected with future surveys like DES [10], BOSS [11] or the satellite project EUCLID [12].

In this brief paper, we are not concerned to use the most sophisticated or realistic model of biasing, we mainly want to make a point of principle. In subsequent work we shall investigate the ideas presented here using a more realistic biasing scheme e.g. the one studied in Ref. [13]. In the next section we determine the biased correlation function and power spectrum for a simple non-linear biasing model and discuss the main features. In section 3 we conclude.

2. The biased power spectrum

Non-linear Newtonian clustering does not induce an inverse cascade, but rather moves power from large to smaller scales (direct cascade). This effect flattens the density power spectrum on small scales which according to linear perturbation theory should behave like $P(k) \propto k^{-3}\ln^2(k)$ to a behavior roughly like $P(k) \approx k^{-2}$ found in numerical simulations, see e.g. [2]. On large scale, inflation predicts that $P(k) \propto k^n$, and the spectral index $n = 0.963 \pm 0.014$ has been measured with the WMAP satellite [8]. This is a very special power spectrum. Since the power spectrum is the Fourier transform of the correlation function,

$$P(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi(r) j_0(kr) r^2 dr\,, \quad j_0(x) = \frac{\sin x}{x}\,, \tag{2.1}$$

j_0 is is the spherical Bessel function of zeroth order. $P(0) = 0$ implies

$$\int_0^\infty \xi(r) r^2 dr = 0\,. \tag{2.2}$$

This integral constraint is very unusual and implies that the matter distribution represents a 'super-homogeneous' system in a precise statistical sense [14, 15].

For example, this requires that there is absolutely no white noise in the system because this would add a constant to the power spectrum. From Eq. (2.2) it is clear that if biasing is not everywhere linear, leading to a galaxy correlation function which is not simply $\xi_g(r) = b\xi(r)$, there is a high chance that the very subtle and non local integral constraint (2.2) will be violated and $P_g(0) \neq 0$.

We illustrate this point by using the simple biasing scheme which has been proposed by Kaiser [3]: be $\delta(\mathbf{x})$ the density fluctuations with mean $\langle \delta(\mathbf{x})^2 \rangle = \sigma^2$. We assume that galaxies form when the density fluctuation is larger than a threshold ν, $\delta(\mathbf{x}) > \nu\sigma$. The correlation function of galaxies forming after this prescription is given by the correlation function of the threshold sets θ_ν defined by [3, 4]

$$\theta_\nu(\mathbf{x}) \equiv \theta(\delta(\mathbf{x}) - \nu\sigma) = \begin{cases} 1 \text{ if } & \delta(\mathbf{x}) \geq \nu\sigma \\ 0 \text{ else.} \end{cases} \tag{2.3}$$

$\langle \theta(\mathbf{x}) \rangle = \langle \theta(\mathbf{x})^2 \rangle = Q(\nu)$ gives the fraction of the volume in which $\delta(\mathbf{x}) > \nu\sigma$. By construction, the amplitude of θ_ν never exceeds 1 and therefore also its correlation function,

$$\xi_\nu(r) = \langle \theta_\nu(\mathbf{x})\theta_\nu(\mathbf{y}) \rangle \leq 1 \qquad r = |\mathbf{x} - \mathbf{y}|\,.$$

This overall normalization is arbitrary and so is the overall normalization of its power spectrum P_ν. We shall therefore not comment on the overall amplitude of the power spectrum in the following. It can be adjusted at will.

If we assume that $\delta(\mathbf{x})$ is a Gaussian field with vanishing mean, the one point distribution is

$$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}} ,$$

and the 2-point function is given by

$$P(\delta_1, \delta_2, r) = \frac{1}{2\pi\sqrt{\sigma^4 - \xi(r)^2}} \exp\left(-\frac{\sigma^2(\delta_1^2 + \delta_2^2) - 2\xi(r)\delta_1\delta_2}{2(\sigma^4 - \xi^2(r))} \right) , \tag{2.4}$$

where $\delta_1 = \delta(\mathbf{x}_1)$, $\delta_2 = \delta(\mathbf{x}_2)$ and $r = |\mathbf{x}_1 - \mathbf{x}_2|$. From this it is easy to derive [3, 6] that the correlation function for the biased field θ_ν is given by

$$\xi_\nu(r) = \frac{\int_\nu^\infty dx e^{-x^2/2} \int_{\mu(r)}^\nu dy e^{-y^2/2}}{[\int_\nu^\infty dx e^{-x^2/2}]^2} , \tag{2.5}$$

where $\mu(r) = (\nu\sigma^2 - \xi(r)x)/\sqrt{\sigma^4 - \xi^2(r)}$. Clearly, if $\xi(r) = 0$, $\mu(r) = \nu$ and hence also $\xi_\nu(r) = 0$. Furthermore, it can be shown that $\xi_\nu(r) \simeq \nu^2\xi(r)$ in the regime where $\xi(r) \ll \sigma^2$ and $\xi_\nu(r) \ll 1$. However, as has been pointed out in Ref. [6], the biased power spectrum,

$$P_\nu(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi_\nu(r) j_0(kr) r^2 dr \tag{2.6}$$

does no longer vanish at the origin,

$$P_\nu(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi_\nu(r) r^2 dr \neq 0 . \tag{2.7}$$

2.1 Very large scales

We have calculated the biased power spectrum $P_\nu(k)$ from an underlying ΛCDM matter power spectrum as approximated in [16] for different values of the biasing parameter ν. This calculation has already been performed in Ref. [6], but there an exponential cutoff on small scales has been introduced for convenience. Here we use the full spectrum given in Ref. [16] which does, however neglect baryons and makes use of the fitting formula obtained in [5] for the transfer function:

$$T(x) = \frac{\ln(1 + 0.171x)}{0.171x} \left[1 + 0.284x + (1.18x)^2 + (0.399x)^3 + (0.490x)^4\right]^{-1/4} \tag{2.8}$$

with $x \equiv k/k_{eq}$ and $k_{eq} = \sqrt{2(1 + z_{eq})\Omega_M} H_0$. The matter power spectrum is given in terms of the transfer function by $P(k) = T^2(k)P_0(k)$, with $P_0(k) \propto k^n$ the primordial power spectrum.

In Fig. 1 we show the biased power spectrum obtained from the underlying ΛCDM matter power spectrum. One clearly sees that, even though the underlying spectrum behaves like k^n at large scales (small k), the biased spectra tend to a constant since the integral constraint (2.2) is violated, i.e., there is no longer an exact cancellation between correlations at small scales and anti-correlations at large scales. It is interesting to note that the modification of the power spectrum can be split into two effects: a universal

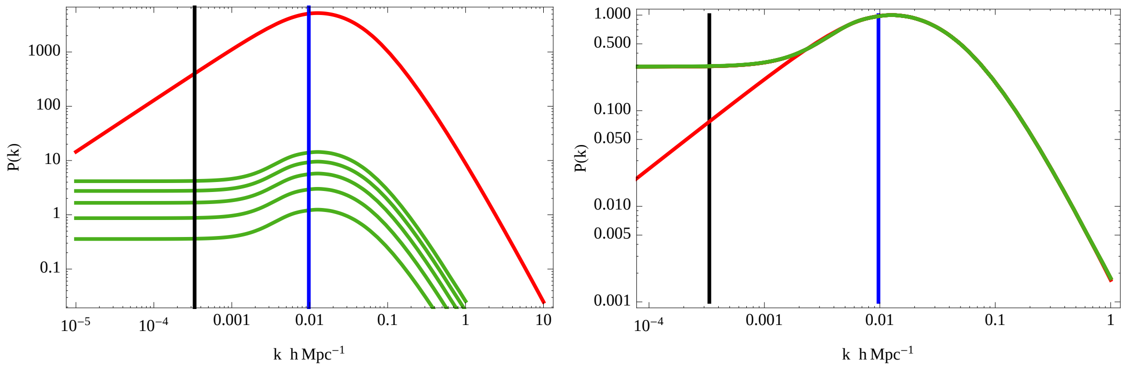

Figure 1: In the left panel we show the underlying ΛCDM power spectrum (red line) and the corresponding biased power spectra (green lines) for $\nu = 5$, 4, 3, 2 and 1 from top to bottom. In the right panel we have normalized the power spectra to their maximum value after which they all collapse to $P_1(k)$ as discussed in the main text. We have also indicated the scales corresponding to the horizon scale at equality, k_{eq} (blue vertical line), and the present horizon scale, k_0 (black vertical line).

modification in the shape and a vertical shift (a reduction of power in the present case) that depends on the biasing parameter ν. This becomes apparent when we normalize the biased spectra to their maximum value. As shown in Fig 1, the normalized biased power spectra share the same shape irrespectively of the particular value of the parameter ν. Moreover, we see that the only modification when compared to the underlying ΛCDM spectrum is the appearance of the plateau for very large scales, spoiling the $P(k) \propto k^n$ behaviour. Thus, we can conclude that the biased power spectrum can be factorized as $P_\nu(k) = b_1(\nu)P_1(k)$, with $b_1(\nu)$ accounting for the ν-dependent shift and $P_1(k)$ the universal biased power spectrum shown in Fig. 1 that becomes a constant for large scales and tends to the underlying power spectrum $P(k)$ on small scales. Taking advantage of this last property, we obtain the following analytical expression for the shift function:

$$b_1(\nu) = \frac{\pi}{2\sigma^2}\frac{e^{-\nu^2}}{\left(1 - \mathrm{erf}(\nu/\sqrt{2})\right)^2} \tag{2.9}$$

where $\mathrm{erf}(x)$ denotes the error function. In Ref. [6] it is derived that $b_1(\nu)\xi(r) \simeq \xi_\nu(r)$ in the regime where $\nu\xi(r) < 1$ and $\xi(r)/\sigma^2 \ll 1$, hence on sufficiently large scales. Here we see that $b_1(\nu)P(k)$ is a good approximation to the power spectrum on *small* scales, large k.

In the right panel of Fig. 1, we compare the normalized biased power spectra which all collapse on one line with the underlying ΛCDM power spectrum. Contrary to the standard believe, the Kaiser biasing scheme leads to linear bias of the power spectrum on *small* scales and non-linear bias on large scales.

In Fig. 2 we show the results of the same calculation but the underlying power spectrum has been smoothed with a Gaussian window function. In real observations, the correlation function has to be smoothed over some scale R e.g. with a Gaussian window

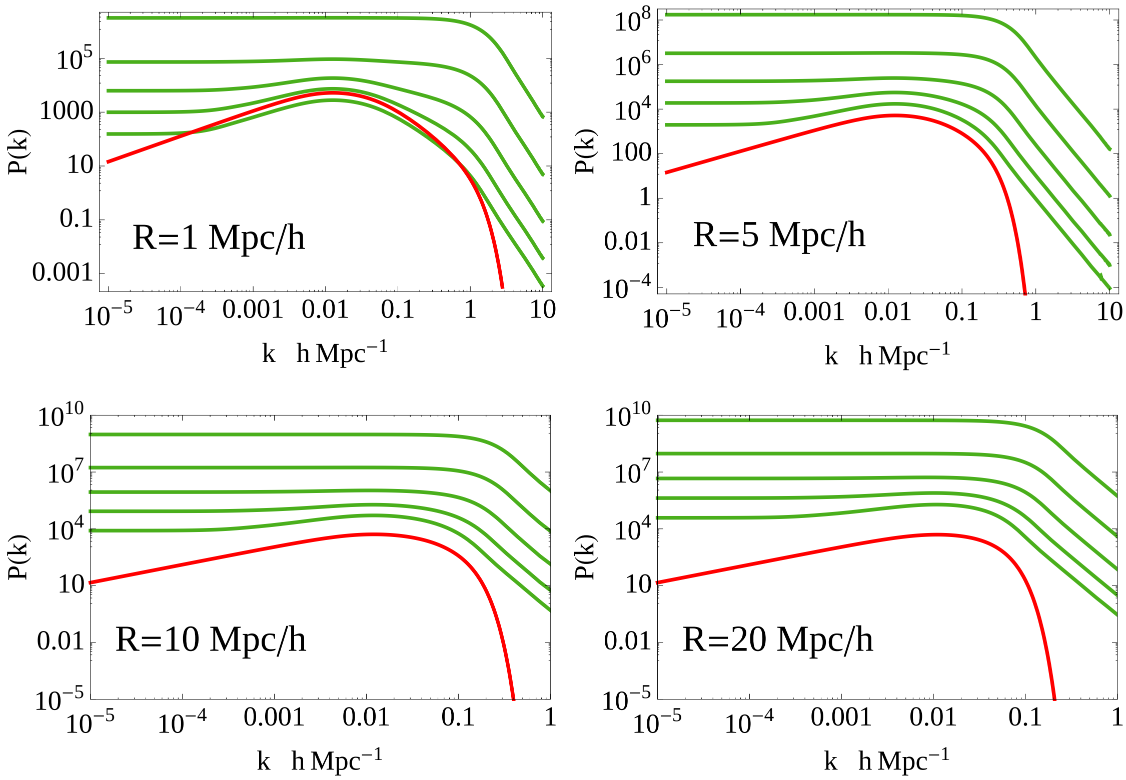

Figure 2: We show the biased power spectrum for a smoothed underlying power spectrum as explained in the text for several smoothing scales (indicated in the corresponding panels) and for ν =5, 4, 3, 2 and 1 from top to bottom.

function $W(r/R)$. Here we assume that also the biasing has to be applied to the smoothed correlation function. The smoothed underlying power spectrum is

$$P = P(k,R) = |\hat{W}(kR)|^2 P(k) \tag{2.10}$$

where $\hat{W}(kR) = \exp(-R^2k^2/2)$ is the Fourier transform of the normalized window function $W(\mathbf{x}/R) = \frac{1}{V_R}\exp[-\mathbf{x}^2/(2R^2)]$. In this case, the shape of the biased power spectrum is no longer universal, but it becomes distorted at all scales. Indeed, for high values of ν, or large smoothing scale the turnover that is present in the underlying power spectrum disappears completely. This disappearance happens for smaller values of ν as the smoothing scale increases. These results are in line with those obtained in Ref. [6] since the smoothing applied here is equivalent to the exponential cutoff for small scales introduced that work.

2.2 The baryon acoustic oscillations

We now take into account the baryons and study the effects of the considered biasing scheme on the BAO's. For that, we shall use the fitting formula given in [17] for the transfer function. Note that the effect of introducing baryons is not only the appearance of the BAO's, but also a suppression on small scales (Silk damping [1]).

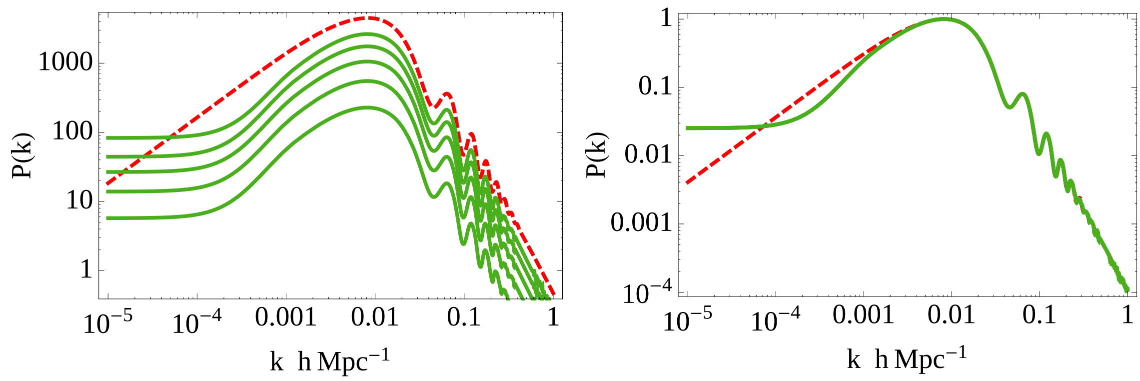

Figure 3: Biasing in the presence of baryons. For this plot we choose $\Omega_b h^2 = 0.025$, $\Omega_M h^2 = 0.05$ and, $h = 0.5$. With this too large ratio of Ω_b/Ω_M the BAO's are well visible. In the left panel we show the underlying ΛCDM power spectrum (red dashed line) that includes the effects of baryons and the corresponding biased power spectra (green solid lines) for $\nu = 5$, 4, 3, 2 and 1 from top to bottom. In the right panel we have normalized the power spectra to their maximum value that clearly shows the two effects explained in the main text.

In Fig. 3 we show the obtained results for this case. In the plot we use a too small value of $\Omega_M = 2\Omega_b = 0.1$ and $h = 0.5$ in order to make the BAO's well visible. We have performed the calculation also for realistic values of the cosmological parameters and found the same results: the biased power spectrum flattens at the largest scales whereas the shape of the small scales part of the spectrum remains unaffected and, therefore, the BAO's are not altered. Once again, this becomes more apparent when we normalize the spectra to their maximum value as shown in the right panel of Fig. 3. As before, this result implies that the biased power spectra can be factorized as $P_\nu(k) = b_1(\nu)P_1(k)$ with $b_1(\nu)$ the function given in (2.9) and P_1 the universal shape of the biased power spectrum (which of course is different from the one without baryons). However, we do find a difference here with respect to the case without baryons: the power spectrum flattens at a larger scale and, in addition, its decay towards the white noise plateau on large scales is steeper than the $P(k) \propto k$ behaviour of the underlying power spectrum.

Also for this case, we have studied the effect of smoothing with a Gaussian window function. The corresponding results are shown in Fig. 4. As before, the shape of the power spectrum is not conserved after biasing and the turnover tends to disappear for high values of the biasing parameter ν and for large smoothing scale. It is interesting to note that, unlike in the non-smoothed case, the BAO's are distorted by smoothing and they are completely washed out for high values of ν. We find it interesting that smoothing over a scale of only $1h^{-1}$Mpc already has such a significant effect on the power spectrum on large scales.

3. Conclusions

In this work we have studied the effects of a simple biasing scheme on the matter power

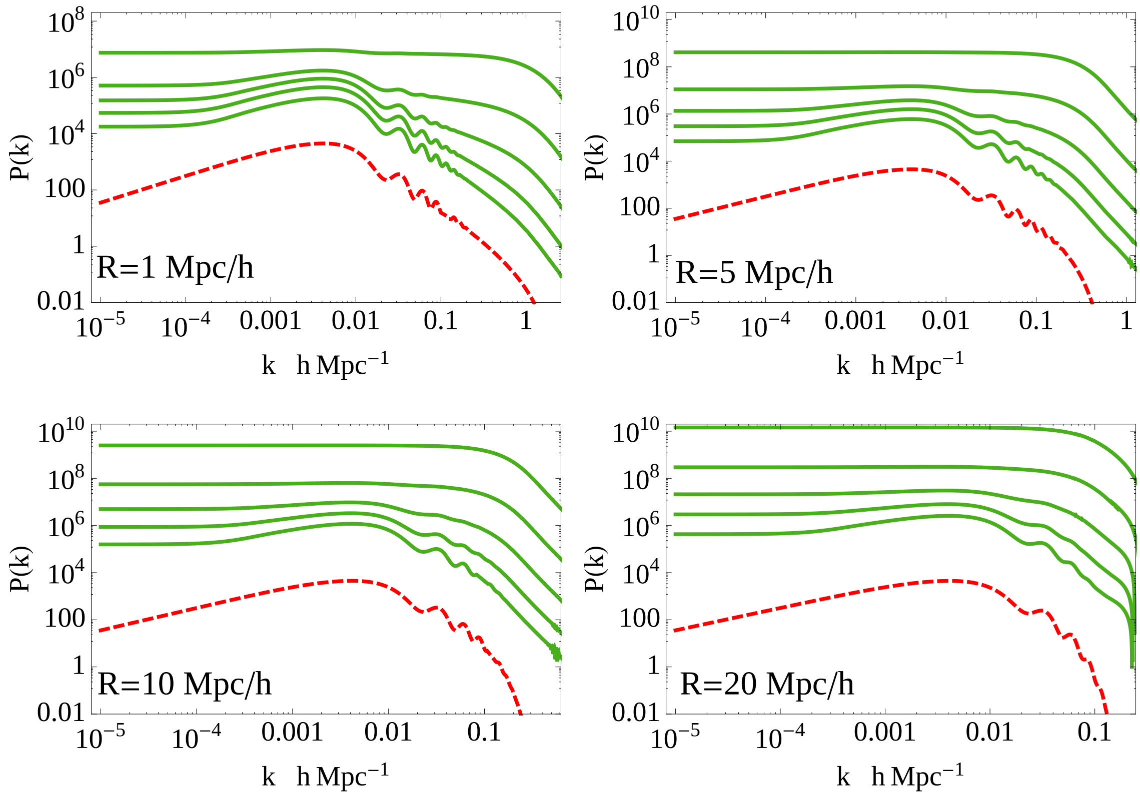

Figure 4: The parameters are like for Fig. 3. We show the underlying ΛCDM power spectrum smoothed with a Gaussian window function (red dashed line) and the corresponding biased power spectra (green solid lines) for $\nu = 5$, 4, 3, 2 and 1 from top to bottom for the different smoothing scales indicated in the panels.

spectrum which has been considered before in Ref. [6]. However, we have used more realistic underlying power spectra, unlike in Ref. [6] where an unrealistic cutoff is introduced. Furthermore, we have studied the modification of the BAO's due to biasing.

First, we have used the approximate BBKS power spectrum, which neglects baryons, and we have found that the biased power spectrum is modified by two effects: a distortion at large scales where the power spectrum flattens and a vertical shift given by (2.9). Contrary to the standard claim, there is linear bias on small scales, large k and a non-linear (k-dependent) modification of the power spectrum on large scales. We have also studied the effect of biasing on a power spectrum smoothed with a Gaussian window function. In that case the biased power spectrum becomes distorted at all scales and the turnover tends to disappear for high values of the biasing parameter or large smoothing scale. Already at a smoothing scale of $1h^{-1}$Mpc, the turnover on large scales nearly disappears for $\nu > 2$.

Finally, we have included baryons and studied the possible effects on the BAO's. We have found the same factorization as in the case without baryons is possible, i.e., the biased power spectrum flattens at large scales, and is simply re-scaled by the constant factor $b_1(\nu)$

at small scales. There are no effects on the BAO's. They are not smeared out by Kaiser biasing. However, the plateau appears at a somewhat larger scale than in the case without baryons and the transition from the peak to the plateau is steeper than the linear slope of the underlying ΛCDM power spectrum. However, the introduction of smoothing distorts the biased power spectrum at all scales. In particular, the BAO's completely disappear for high values of the biasing parameter, $\nu > 3$.

Our results show that, for the biasing scheme considered in this work, the $P \propto k$ slope is in principle observable for nearly a decade before the 'white noise' contribution sets in. This result is less pessimistic than in Ref. [6], but it has to be checked with a more realistic biasing scheme that will be considered in a future project.

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