The dipole of the luminosity distance: a direct measure of $H(z)$

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We show that the dipole of the luminosity distance is a useful observational tool which allows to determine the Hubble parameter as function of redshift, $H(z)$. We determine the number of supernovae needed to achieve a given precision for $H(z)$ and to distinguish between different models for dark energy. We analyse a sample of nearby supernovae and find a dipole consistent with the CMB at a significance of more than 2σ.

One of the biggest cosmological surprises in recent years was the discovery that the Universe is presently undergoing a phase of accelerated expansion [1]. The reason for this behaviour is still a complete mystery. If the Universe is homogeneous and isotropic on large scales, all contributions to the cosmological energy momentum tensor are characterised by their energy density $\rho(z)$ and pressure $P(z)$ as functions of cosmic redshift z. Accelerated expansion requires that overall $\rho + 3P < 0$ today. This can be achieved by introducing a so-called 'dark energy' component with a very negative pressure in addition to the usual pressureless matter. One of the main challenges of observational cosmology is to characterise the properties of this dark energy. The homogeneous and isotropic aspects of dark energy are completely determined by the equation of state parameter $w_{de}(z) \equiv P_{de}(z)/\rho_{de}(z)$ which links its pressure and energy density. The primary goal of observational dark energy studies is to measure the function $w_{de}(z)$.

Current experiments probing the dark energy equation of state measure luminosity distances to supernovae or the angular diameter distance to the last scattering surface via the CMB peak positions. These distances are linked to $w_{de}(z)$ through a double integration, which renders them rather insensitive to rapid variations of the equation of state. The required modelling can lead to strong biases that are difficult to detect and quantify [2]. A direct measurement of the Hubble parameter $H(z)$ would facilitate the derivation of $w_{de}(z)$ immensely and allow for a more direct comparison with model predictions. As an explicit illustration, let us consider a flat universe. The Friedmann equations then yield the following relation between $w_{de}(z)$ and $H(z)$

$$w_{de}(z) = \frac{3}{2}\left(\frac{2/3H'(z) + H^2(z)}{H^2(z) - H_0^2\Omega_m(1+z)^3}\right) . \quad (1)$$

Apart from $H(z)$ and its derivative only the parameter combination $\Omega_m H_0^2$ appears, which will be measured by the Planck satellite to an accuracy of about 1%.

It is possible to obtain $H(z)$ by computing the numerical derivative of the distance data [3], but the current data leads to a very noisy result. In the future, radial baryon oscillation measurements should be able to measure $H(z)$ directly. Here we propose an alternative, completely independent method based on luminosity distance measurements.

In a previous paper [4] we have considered the luminosity distance d_L as function of the source redshift z and direction $\mathbf{n}$. We have shown that not only the direction-averaged luminosity distance,

$$d_L^{(0)}(z) = \frac{1}{4\pi}\int d\Omega_\mathbf{n} d_L(z,\mathbf{n}) = (1+z)\int_0^z \frac{dz'}{H(z')} , \quad (2)$$

but also its directional dependence, $d_L(z,\mathbf{n})$ can be of cosmological interest. The directional dependence can be expanded in terms of spherical harmonics, leading to observable multipoles, $C_\ell(z)$. In this *letter* we concentrate on the dipole (corresponding to $\ell = 1$),

$$d_L^{(1)}(z) = \frac{3}{4\pi}\int d\Omega_\mathbf{n}(\mathbf{n}\cdot\mathbf{e})d_L(z,\mathbf{n}) . \quad (3)$$

Here $\mathbf{e}$ is a unit vector denoting the direction of the dipole and $d_L^{(1)}(z)$ is its amplitude.

As we have discussed in [4] (see also [5]), for $z \gtrsim 0.02$ the dipole is dominated by the peculiar velocity of the observer for all redshifts. The lensing contribution to C_1 is of the order of 10^{-9} while our peculiar motion induces a dipole of $C_1 \simeq 10^{-3} - 10^{-6}$ for $z \lesssim 2$. At high ℓ's, i.e. small scales the lensing contribution dominates for $z \gtrsim 1$, but this does not interfere with the dipole as it averages to zero under the integration (3). Neglecting multipoles higher than the dipole we can write the full luminosity distance as

$$d_L(z,\mathbf{n}) = d_L^{(0)}(z) + d_L^{(1)}(z)(\mathbf{n}\cdot\mathbf{e}) . \quad (4)$$

To derive a formula for $d_L^{(1)}$ (more details are found in [4]), we use the luminosity distance to a source emitting photons at conformal time η in an unperturbed Friedmann universe, $d_L^{(0)} = (1+z)(\eta_0 - \eta)$. The motion of the observer gives rise to a Doppler effect which

is the dominant contribution to the dipole,

$$d_L(\eta, \mathbf{n}) = d_L^{(0)}(\eta)\left[1 - (\mathbf{n} \cdot \mathbf{v}_0)\right] \; , \qquad (5)$$

where $\mathbf{v}_0$ is our peculiar velocity. However, conformal time η is not an observable quantity, but the source redshift, $z = \bar{z}(\eta) + \delta z$ is. Here $\bar{z}(\eta) = 1/a(\eta)$ is the unperturbed redshift. To first order

$$d_L(\eta, \mathbf{n}) = d_L(\bar{z}, \mathbf{n}) = d_L(z, \mathbf{n}) - \frac{d}{d\bar{z}}d_L^{(0)}(\bar{z})\delta z \; . \qquad (6)$$

With $d_L^{(0)}(\bar{z}) = (1 + \bar{z})(\eta_0 - \eta)$, we have

$$\frac{d}{d\bar{z}}d_L^{(0)} = (1 + \bar{z})^{-1}d_L^{(0)} + \mathcal{H}^{-1} \qquad \text{and}$$
$$\delta z = -(1 + \bar{z})(\mathbf{v}_0 \cdot \mathbf{n}) + \text{ higher multipoles} \; . \, (7)$$

Here $\mathcal{H}(z) = H(z)/(1 + z)$ is the co-moving Hubble parameter. Inserting this in Eq. (5), we obtain

$$d_L^{(1)}(z)(\mathbf{n} \cdot \mathbf{e}) = \frac{1 + z}{\mathcal{H}(z)}(\mathbf{n} \cdot \mathbf{v}_0) \; . \qquad (8)$$

Although $\mathbf{v}_0$ is in principle a random variable, we can measure it directly from the CMB dipole which is due to the same motion. Its magnitude is $|\mathbf{v}_0| = (368 \pm 2)$km/s according to the COBE satellite measurements [6]. The amplitude of the luminosity distance dipole is then

$$d_L^{(1)}(z) = \frac{|\mathbf{v}_0|(1 + z)}{\mathcal{H}(z)} = \frac{|\mathbf{v}_0|(1 + z)^2}{H(z)} \; , \qquad (9)$$

and its direction is $\mathbf{e} = \mathbf{v}_0/|\mathbf{v}_0|$. The dipole in the supernova data gives therefore a direct measure of $H(z)$.

As a first step we want to test whether there is a dipole present at all in the supernova distribution, and if its direction and magnitude is compatible with expression (8) (see also [7]). Supernova data is conventionally quoted in terms of magnitudes rather than the luminosity distance. The link between magnitudes and the luminosity distance is given by

$$m - \mathcal{M} = 5 \log_{10}(d_L/10\text{pc}) \qquad (10)$$

where $\mathcal{M}$ is the intrinsic magnitude (however, the absolute magnitude normalisation is degenerate with $\log(H_0)$ and is usually marginalised over). We use the low-redshift sample of 44 supernovae assembled by the SNLS team [8], together with the supernova locations from [9]. To the given photometric error we add a peculiar velocity error of 300 km/s and a constant dispersion of $\Delta m = 0.12$. The latter error ensures a reasonable goodness-of-fit of both the monopole and dipole term. We subtract the monopole of $m(z, \mathbf{n})$ and find the best fit value of $\mathbf{v}_0$ for the dipole. In Fig. 1 we show the angular uncertainty of $\mathbf{v}_0$. The direction is compatible with
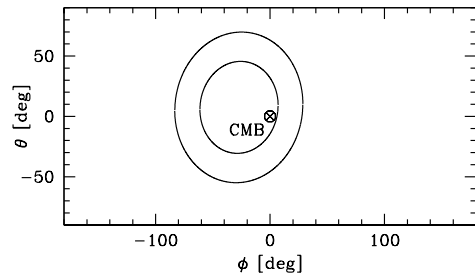

FIG. 1: Direction of the luminosity dipole from a low-redshift supernovae sample, in a celestial coordinate system centred on the CMB dipole (1 and 2σ contours). The two directions agree well.

the CMB dipole at the 1σ level. The magnitude of the luminosity dipole gives $|\mathbf{v}_0| = 405 \pm 192$ km/s, in excellent agreement with the CMB dipole value of 368 km/s.

Fixing the CMB dipole direction and fitting only the amplitude, we obtain $|\mathbf{v}_0| = 358$ km/s with $\chi^2_{\min} = 48.2$ whereas $\mathbf{v}_0 = 0$ gives $\chi^2 = 52.7$. The absence of a dipole is therefore disfavored at over 2σ.

Let us estimate the accuracy with which we can determine $H(z)$ from a measurement of N supernovae in a redshift bin $[z - dz, z + dz]$. We assume that the magnitude of each supernova is known with a precision Δm (independent of z). We consider $\mathbf{v}_0$ given by the CMB dipole and work with the ansatz (4). The error in the magnitude translates into an error on the luminosity distance,

$$\delta d_L(z, \mathbf{n}) = \frac{\ln(10)}{5}d_L(z, \mathbf{n})\delta m(z, \mathbf{n}) \; . \qquad (11)$$

We add the error into our ansatz, setting

$$m(z, \mathbf{n}) = m^{(0)}(z) + m^{(1)}(z)(\mathbf{n} \cdot \mathbf{e}) + \delta m(z, \mathbf{n}) \, (12)$$
$$d_L(z, \mathbf{n}) = d_L^{(0)}(z) + d_L^{(1)}(z)(\mathbf{n} \cdot \mathbf{e}) + \delta d_L(z, \mathbf{n}). \, (13)$$

We assume that different supernovae are uncorrelated, so that the variance of the magnitude is given by

$$\langle \delta m(z, \mathbf{n})\delta m(z, \mathbf{n}')\rangle = 4\pi(\Delta m)^2\delta^2(\mathbf{n} - \mathbf{n}'). \qquad (14)$$

The error on the dipole can now be computed using

$$\delta d_L^{(1)}(z) = \frac{3}{4\pi}\int d\Omega_{\mathbf{n}}(\mathbf{n} \cdot \mathbf{e})\delta d_L(z, \mathbf{n}) \; . \qquad (15)$$

Its variance is

$$\left(\Delta d_L^{(1)}(z)\right)^2 = \left\langle \left(\delta d_L^{(1)}(z)\right)^2 \right\rangle$$
$$= 3\left(\frac{\ln(10)}{5}\right)^2 \Delta m^2 \left(d_L^{(0)}(z)\right)^2 \; .$$
$$(16)$$

As the monopole is much larger than the dipole we have neglected the latter in the previous expression and obtain our final formula for the variance of the dipole

$$\Delta d_L^{(1)}(z) \simeq \frac{\sqrt{3}\ln(10)}{5} d_L^{(0)}(z)\Delta m = \sqrt{3}\Delta d_L(z) \ . \quad (17)$$

The absolute error on the dipole is therefore comparable to the error on the monopole and, not surprisingly, the relative error is much larger,

$$\frac{\Delta d_L^{(1)}(z)}{d_L^{(1)}(z)} = \sqrt{3}\frac{\Delta d_L^{(0)}(z)}{d_L^{(1)}(z)} \gg \frac{\Delta d_L^{(0)}(z)}{d_L^{(0)}(z)} \ . \quad (18)$$

We will therefore need a large number of supernovae to determine $H(z)$ with reasonable accuracy.

As the Hubble parameter is inversely proportional to the dipole, its relative error is simply

$$\frac{\Delta H(z)}{H(z)} = \frac{\Delta d_L^{(1)}(z)}{d_L^{(1)}(z)} = \frac{\sqrt{3}\ln(10)}{5|\mathbf{v}_0|}\frac{d_L^{(0)}(z)H(z)}{(1+z)^2}\Delta m. \quad (19)$$

This formula is valid for any model of dark energy. Once we have measured the luminosity distance, we can calculate the monopole and the dipole, deduce the Hubble parameter, and equation (19) gives the error on $H(z)$ per supernova at that redshift. We plot the relative error on $H(z)$ (which is the same as the relative error on the dipole amplitude) in Fig. 2 for a flat ΛCDM universe with $\Omega_\Lambda = 0.7$. We use two values for the error on the magnitude, $\Delta m = 0.1$ and $\Delta m = 0.15$. This is comparable to the accuracy of recent supernova surveys like SNLS [8].

In the future it may be possible to control systematic errors much better – indeed the very dipole that we want to measure is part of the systematic error budget of current surveys [10]! Proper assessment of the dipole contribution may therefore also help to measure the monopole with higher precision. Our assumed value for Δm is probably pessimistic as most systematic uncertainties are expected to affect the overall luminosity at a given redshift, i.e. the monopole. The dipole which relies on the angular distribution of the luminosity should be far more resistant to effects like for example evolution of the supernova population with redshift.

Observing N independent supernovae, the mean error on the magnitude is reduced to $\frac{\Delta m}{\sqrt{N}}$. In Fig. 3 we plot the number of supernovae needed to measure $H(z)$ with an accuracy of 30 %. This number scales quadratically with the errors; we need to measure 100 times more supernovae to decrease the error by a factor of 10 to 3%. On the other hand, if we manage to decrease Δm by a factor of 10 through an improved understanding of supernova explosions and better measurements, then we need 100 times fewer supernovae.

One crucial question about dark energy is whether it does indeed behave as a cosmological constant or not.
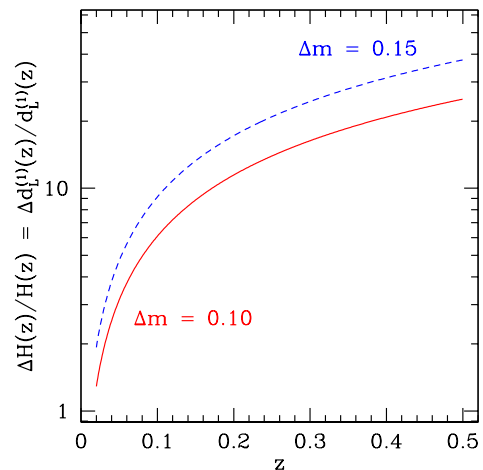

FIG. 2: We show the relative error in $H(z)$ for one supernova, as a function of the redshift, in a flat ΛCDM universe with $\Omega_\Lambda = 0.7$ and for two different values of the intrinsic error $\Delta m = 0.1$ and $\Delta m = 0.15$. This represents as well the relative error in the dipole $d_L^{(1)}(z)$.
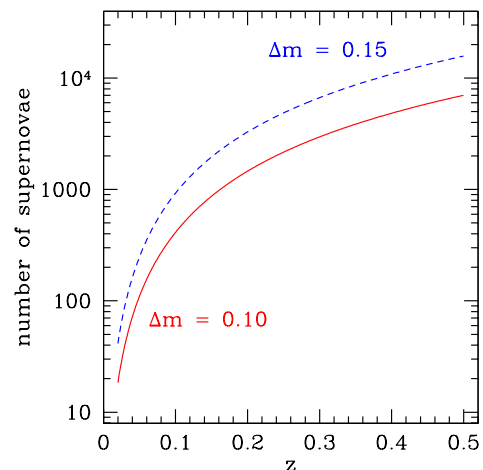

FIG. 3: We show the number of supernovae needed to measure $H(z)$ with an accuracy of 30 %, in a flat ΛCDM universe with $\Omega_\Lambda = 0.7$, as a function of the redshift and for two different values of the intrinsic error $\Delta m = 0.1$ and $\Delta m = 0.15$.

Having measured the dipole at different redshifts, it is possible to compare directly the measured value of $H(z)$ with the one predicted for a flat ΛCDM universe. If the two do not agree, dark energy must be due to a different mechanism, like quintessence or a modification of general relativity. In Fig. 4 we plot the number of supernovae needed to distinguish the two cases, by demanding that the difference $|H(z) - H_{\Lambda\text{CDM}}(z)|$ be larger than the error $\Delta H(z)$. For comparison, the relative difference between the Hubble parameter in a flat pure CDM universe and in a flat ΛCDM universe with $\Omega_\Lambda = 0.7$ is 10% at $z = 0.1$, 19% at $z = 0.2$ and 27% at $z = 0.3$.

Our method tests directly the expansion speed of the universe at all the redshifts where we measure the luminosity distance dipole. Any deviation in $H(z)$ from theoretical predictions will be immediately detected. If we measure only the usual monopole of the luminosity distance then a well-localised deviation may easily be smeared out and lost by the additional integration.
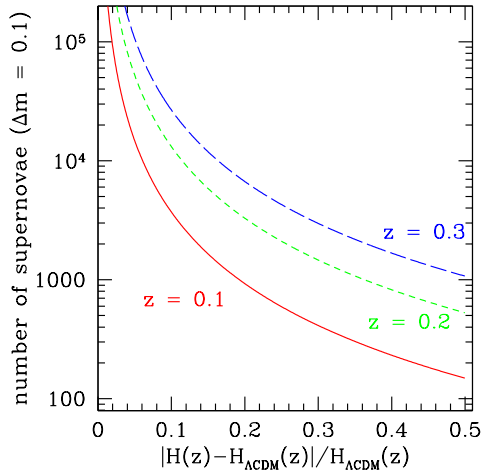

FIG. 4: We show the number of supernovae needed to differentiate the measured Hubble parameter $H(z)$ from the theoretical one in a flat ΛCDM universe $H_{\Lambda CDM}(z)$, as a function of the relative difference $\frac{|H(z)-H_{\Lambda CDM}(z)|}{H_{\Lambda CDM}(z)}$, for redshifts $z = 0.1$, $z = 0.2$ and $z = 0.3$ and intrinsic error $\Delta m = 0.1$.

Looking at the figures it is readily apparent that accuracy is best at low redshift. This is not necessarily a drawback, as dark energy is expected to dominate at low redshift and so is best observed in this redshift range. However, below $z \stackrel{<}{\sim} 0.04$ non-linear effects probably become important which may lead to systematic effects in the distribution of supernovae. Future baryon oscillation surveys will primarily target higher redshifts, $z \stackrel{>}{\sim} 0.5$, where they reach maximum sensitivity. The angular distribution of low-redshift supernovae is therefore a complementary probe. Also the number of supernovae needed is not completely unrealistic. Very large surveys which plan to measure of the order of 10^4 to 10^5 supernovae are presently discussed [11].

As a final remark, even though uniform sky coverage is not essential, a survey designed to measure the dipole should optimally cover a large part of the sky. If we only observe supernovae in one small patch, it may be difficult to extract the dipole without contamination from lensing which dominates over the dipole for $\ell \stackrel{>}{\sim} 100$ and $z \stackrel{>}{\sim} 1$ (see [4]). If possible, the observed supernovae should cover the regions of the sky aligned and anti-aligned with

the CMB dipole where the luminosity difference is maximal.

In this *letter* we have discussed a novel method for measuring directly the expansion history of the Universe. We have shown that the dipole of the supernova distribution on the sky is proportional to $1/H(z)$. It is therefore possible to extract *directly* $H(z)$ from the dipole. This is advantageous compared to the monopole of the luminosity and angular diameter distance which measure only the integral over the Hubble parameter. With a present data set of 44 low redshift supernovae we have measured the dipole and it is in good agreement with the CMB.

We have also discussed the accuracy with which we can measure the Hubble parameter, and found that we need a large number of supernovae. However, future planned surveys are expected to deliver these. Given that most surveys concentrate on high-redshift supernovae while the dipole is most useful at moderate redshifts, $z \stackrel{<}{\sim} 0.5$, it may be preferable to propose a dedicated low-z supernova survey.

Finally, the dipole is a quantity independent of the monopole. Given a survey with a sufficient number of supernovae it is possible to measure both. This improves the measurement of the dark energy properties and additionally serves as a cross-check for systematic errors.

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