Is Cosmology Compatible with Blue Gravity Waves ?

Roberta Camerini
Physics Department, Universita' di Roma "La Sapienza, Ple Aldo Moro 2, 00185, Rome, Italy

Ruth Durrer*
Département de Physique Théorique, Université de Genève,
24 quai Ernest Ansermet, 1211 Genève 4, Switzerland.

Alessandro Melchiorri†
Physics Department and INFN, Universita' di Roma "La Sapienza, Ple Aldo Moro 2, 00185, Rome, Italy

Antonio Riotto‡
CERN, Theory Division, Genève 23, CH-1211, Switzerland
INFN, Sezione di Padova, via Marzolo 8, 35131 Padova, Italy
(Dated: February 11, 2008)

A primordial gravitational wave background with positive (blue) spectral index is expected in several non-standard inflationary cosmologies where the stress-energy tensor violates the null energy condition. Here we show that a sizable amount of blue gravitational waves is compatible with current cosmological and astrophysical data. So far most of the works on parameter estimation from cosmic microwave background data have assumed a negative or negligible spectral index. The present limits on cosmological parameters, especially on the scalar spectral index, widen up considerably when one allows also for blue tilts of the tensor spectrum. Since the amplitude of the CMB B-mode polarization is larger in these models, future data from Planck are likely to provide crucial measurements.

PACS numbers: 98.80.-k 95.85.Sz, 98.70.Vc, 98.80.Cq

I. INTRODUCTION

During the last decade or so, the fluctuations and the polarization of the cosmic microwave background (CMB) have proven to be the most valuable observational tool to constrain cosmological models (see e.g. [1]). To a big extent this is due to that fact that CMB physics is sufficiently simple so that it can be calculated to high accuracy with moderate computational investment. Therefore, accurate experimental results can be compared with accurate theoretical predictions. CMB observations are so valuable, since they provide a window to the physics at inflation (see e.g. [2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13]).

This physics most probably involves the highest energies ever probed by an experiment, more the ten orders of magnitude higher than energies achieved in terrestrial experiments like LHC at CERN. An important prediction of simple inflationary models is the production of a stochastic background of gravity waves ([14]) with a slightly tilted spectrum,

$$n_T = -2\epsilon \ , \tag{1}$$

where $\epsilon = -\dot{H}/H^2$ denotes a slow roll parameter from inflation and H is the Hubble rate during the inflationary stage. Since in standard inflation ϵ is strictly positive [15], in the usual parameter estimation routines, the tensor spectral index is assumed to be "red" ($n_T \leq 0$, see e.g. [3, 11]) or negligible [1]. However, this does not rule out a priori the possibility that the spectral index of tensor modes might be positive, $n_T > 0$, i.e. "blue". For instance, in the string gas cosmology set-up [16], where scalar metric perturbations are thought to originate from initial string thermodynamic fluctuations [17], a spectrum of blue gravity waves (BGW hereafter) is predicted [18]. The latter is also a generic prediction of a class of four-dimensional models characterized by a bouncing phase of the universe. To induce the bounce, the stress-energy tensor must violate the null energy condition (NEC). In a spatially flat FRW metric, the NEC corresponds to the inequality $\dot{H} < 0$ and is ultimately responsible for the red tensor spectrum in standard inflation. In the class of bouncing models [19] the scalar metric perturbations are originally of isocurvature nature and they are subsequently transformed into adiabatic ones. The violation of the NEC allows a BGW spectrum. The same is true in the so-called super-inflation models [20] where inflation is driven by a component violating the NEC. BGW may occur also in scalar-tensor theories and $f(R)$ gravity theories.

While we are aware that all the scenarios mentioned so far are not free from criticisms [21, 22], due to our ignorance of the dynamics of the very early universe, in this paper we will assume a more phenomenological attitude accepting the possibility that tensor modes might have a

---

*Electronic address: ruth.durrer@physics.unige.ch
†Electronic address: alessandro.melchiorri@roma1.infn.it
‡Electronic address: riotto@cern.ch

blue spectrum. This option, together with the rather surprising fact that, as far as we know, this parameter range has not yet been fully analyzed, prompted us to investigate whether the presence of such a BGW spectrum is compatible with current cosmological and astrophysical observations[1]. Moreover, a BGW spectrum can affect the constraints on other cosmological parameters which have recently been derived from present observations of the cosmic microwave background. It is not surprising that the limit on the tensor to scalar ratio r depends on this assumptions, but as we shall see below, also other quantities like the scalar spectral index, n_s, the physical baryon density $\Omega_b h^2 = \omega_b$ and the cold dark matter density, $\Omega_c h^2 = \omega_c$ do. (Here Ω_b is the baryon density parameter, Ω_c is the cold dark matter density parameter and h is the value of the Hubble constant in units of 100km/sec/Mpc.)

A careful study of the current astrophysical constraints on the spectral index of the gravity wave background has been recently presented in [23]. If we assume that the stochastic gravity wave spectrum which affects CMB fluctuations, at wave lengths of the order of the Hubble scale from $H_0^{-1} \simeq 3 \times 10^{17} h^{-1}$sec to about $10^{-2} H_0^{-1}$, extends up to wavelengths of about 10^9sec relevant for the timing of millisecond pulsars, constraints of the order of $n_T \lesssim 0.53$ can be found [23, 24, 25]. Similar constraints can be obtained from the LIGO interferometer (see [26]) while applying the nucleosynthesis bound on a gravity wave background yields the best constraint, $n_T \lesssim 0.15$. However, these limits have been obtained by assuming the CMB upper limit on the scalar/tensor ratio of primordial perturbations (taken at wavelength $k = 0.002 hMpc^{-1}$) $r < 0.3$ taken from the recent WMAP+SDSS analysis of [1] which assumes $n_T = 0$. Since a correlation clearly exists between the tensor amplitude and the spectral index (e.g., clearly no constraint on n_T can be derived if r is negligible), here we provide a further analysis by properly analyzing the full CMB data and correlations. Moreover, those limits apply only if the rather bold extrapolation is made that the gravity wave spectrum has a constant spectral slope n_T over the range of many orders of magnitude. The wavelength relevant for LIGO is $\sim$ 10sec and for nucleosynthesis even a fixed spectral index up to the Planck scale, $\sim 10^{-43}$sec, is assumed to obtain the above rather stringent limits [23, 26]. Since these extrapolations are so bold (we are not aware of any physical situation where a scaling behavior extends over more than ten orders of magnitude), we shall also study the case where the limits derived in Ref. [23] do not apply.

Below we also show how the limits on other parameters are affected when we allow for BGW spectra. We investigate the 3-year WMAP data [27] and we produce forecasts for the very near future Planck [28] satellite experiment.

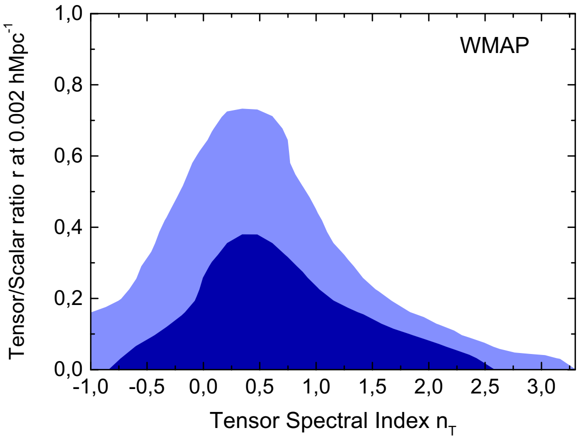

FIG. 1: 68% and 95% likelihood contour plots from WMAP data where no external prior on the spectral index of the gravity wave background is assumed.

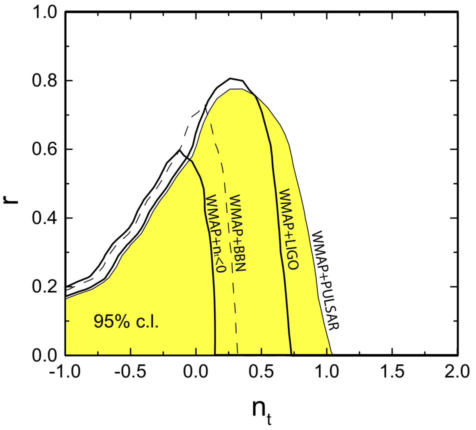

FIG. 2: 95% likelihood contour plots on the n_T-r plane from WMAP data when external prior on the spectral index of the gravity wave background are assumed.

II. ANALYSIS METHOD AND RESULTS

The method we adopt is based on the publicly available Markov Chain Monte Carlo package `cosmomc` [29] with a convergence diagnostics done through the Gelman and Rubin statistics. We sample the following eight-dimensional set of cosmological parameters, adopting flat priors on them: the baryon and cold dark matter densities ω_b and ω_c, the ratio of the sound horizon to the angular diameter distance at decoupling, θ_s, the scalar spectral index n_S, the overall normalization of the spectrum A at $k = 0.002$ Mpc^{-1}, the optical depth to reionization, τ, the tensor-to-scalar ratio r at $k = 0.002$ Mpc^{-1} and,

---

[1] A notable exception is the paper [13]. In this work the authors considered a BGW but with an upper limit $n_T < 0.2$ which, as we will see, is much smaller than the range of values studied here.

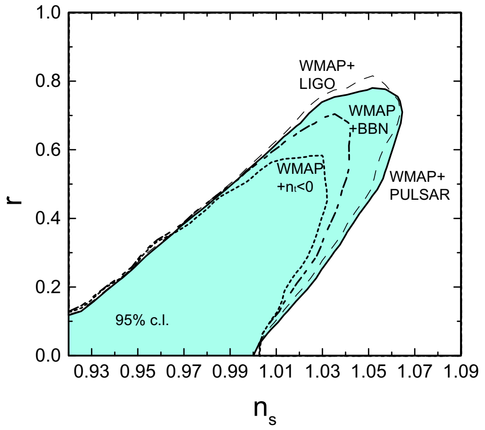

FIG. 3: 95% likelihood contour plots on the n_S-r plane from WMAP data when external prior on the spectral index of the gravity wave background are assumed.

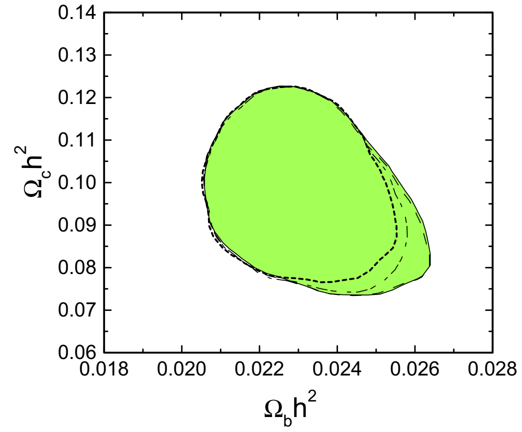

FIG. 4: 95% likelihood contour plots on the ω_b-ω_c plane from WMAP data when external priors on n_T are assumed. The solid line corresponds to the WMAP+PULSAR, the dashed line to WMAP+LIGO, the long-short dashed to WMAP+BBN and the dotted line to WMAP+$n_T < 0$ case.

finally, the tensor spectral index n_T. Furthermore, we consider purely adiabatic initial conditions and we impose spatial flatness.

We include the three–year WMAP data [1] (temperature and polarization) with the routine for computing the likelihood supplied by the WMAP team. Moreover we also include in a separate analysis the new, high quality, fine-scale measurements from the ACBAR experiment ([7]).

We have studied three different possible priors on the combination between n_T and r as derived in Ref. [23] from pulsar timing, LIGO and nucleosynthesis with the Planck scale as the upper cutoff:

$$n_T \leq 0.0477 \log\left(\frac{2.09\times 10^4}{r}\right) \quad \text{PULSAR}, \tag{2}$$

$$n_T \leq 0.0223 \log\left(\frac{1.49\times 10^{10}h^2}{r}\right) \quad \text{LIGO}, \tag{3}$$

$$n_T \leq 0.00714 \log\left(\frac{3.4\times 10^{9}h^2}{r}\right) \quad \text{BBN}. \tag{4}$$

It is important to notice the dependence of the LIGO and BBN constraints on h.

In addition to this astrophysical constraints we have also simply considered a prior $-1 < n_T < 20$ without taking the above limits into account, since it may very well be that the gravity wave spectrum is modified on some scale below those needed for the limits of Eqs. (2) to (4). The results are also compared with a $2-\sigma$ upper limit of $n_T < 0$

A first analysis is made in the theoretical framework described above with no upper limits on n_T. The likelihood contour plot in the n_T-r plane is reported in Figure 1. When only the WMAP data is considered, no upper limit on n_T alone can be derived. The reason is simple: gravity waves with BGW spectra can always be accommodated with the WMAP data by lowering the tensor/scalar ratio r. However it is possible to derive the following 95% upper limits on n_T in function of r: $n_T < 2.1$ for $r > 0.1$ and $n_T < 3.2$ for $r > 0.01$. In the range $0.05 < r < 0.6$ and $0.75 < n_T < 3$ we derive the following upper limit fit at 95% c.l. from WMAP only:

$$r + 0.68n_T - 0.12n_T^2 \leq 1 \tag{5}$$

while including the ACBAR data we obtain:

$$r + 0.41n_T - 0.06n_T^2 \leq 0.7 \tag{6}$$

again at 95% c.l.. Assuming $r > 0.05$ and no prior on n_T, we found from WMAP+ACBAR $-1.51 < n_T < 2.62$ at $95\% c.l.$

These limits, while new and on very different scales, are however rather weak if compared with the upper limits described in Eqs. (2) to (4) above. It is however interesting to notice that even when these stronger limits are considered, BGW spectra are still compatible with CMB observations. Since this contribution has not been usually considered in CMB parameter analysis, it is interesting to study the possible effect of such a BGW background.

In Figure 2, 3 and 4 we show how cosmological parameters as inferred from the WMAP 3-year data are affected by a BGW spectrum.

In Figure 2 we see how current external priors on n_T affect the WMAP bounds on r. As we can see, current bounds on r obtained for $n_T < 0$ can be relaxed by as much as a factor 1.4 when BGW spectra in agreement with LIGO and PULSAR data are assumed. In particular we found the following 95% c.l. upper limits on r: $r < 0.64$, $r < 0.65$, $r < 0.55$ and $r < 0.47$ when the WMAP data is combined with the PULSAR, LIGO, BBN and $n_T < 0$ priors respectively. We found that the constraints with no external prior such that $n_T < 1$ are very similar to the PULSAR and LIGO cases.

In Figure 3 we see how the parameter ranges for the pair (n_S, r) are affected by a blue tilt in the tensor spectrum. First, it is interesting to note that a larger tensor contribution is allowed and the scalar spectral index can then be bluer. This comes from the well known partial degeneracy between rising the scalar spectral index which reduces the Sachs Wolfe plateau if the height of the first acoustic peak is fixed, and at the same time rising the tensor contribution which enhances the Sachs Wolfe plateau. This degeneracy is strengthened if $n_T > 0$ since then tensor fluctuations in the CMB extend to somewhat smaller scales. Disregarding the very speculative nucleosynthesis prior, the 2σ limit on the scalar spectral index is enhanced from $n_s = 0.981 \pm 0.026$ to $n_s = 0.994 \pm 0.032$ when allowing for BGW spectra consistent with pulsar timing. BGW's affect also the constraints on the optical depth parameter, which moves from $\tau = 0.091 \pm 0.030$ to $\tau = 0.097 \pm 0.032$ for the WMAP+PULSAR case.

As one sees in Figure 4, enhancing the scalar spectral index is slightly correlated with enhancing ω_b which leads to stronger Silk damping. The higher baryon density is compensated by a lower cold dark matter density. This not in a way so that $\omega_m = \omega_c + \omega_b$ would remain constant, but so that the total matter density decreases which also reduces the acoustic peaks, to compensate for the blue spectral index. The 2σ limits on ω_c and ω_b change from $\omega_b = 0.0228 \pm 0.0009$ and $\omega_c = 0.100 \pm 0.009$ to $\omega_b = 0.0232 \pm 0.0011$ and $\omega_c = 0.098 \pm 0.01$ respectively when allowing for blue tensor spectra consistent with pulsar timing. The effect of BGW's is therefore smaller on those parameters but still measurable.

While current CMB constraints are clearly affected by the inclusion of a blue gravity wave component it is interesting to investigate the impact on future measurements as those expected from the Planck satellite. The first important observation is that a blue tensor component dramatically enhances the probability for the detection of a gravity wave background with PLANCK. In Figure 5 we plot the best fit angular anisotropy power spectra to the WMAP data for the different external priors on n_T. As we can see, all the models give nearly identical power spectra, i.e. it is impossible to discriminate between those model with current CMB observations.

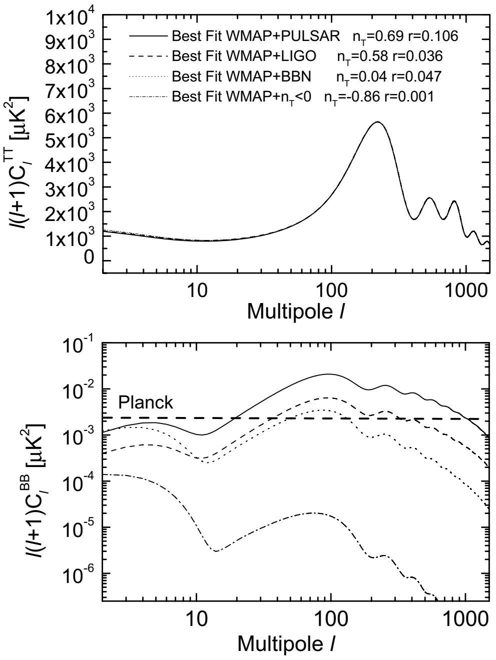

FIG. 5: Top Panel: CMB Temperature anisotropy angular power spectra for the best fit models to current WMAP data for different choices of external priors on n_T. Bottom Panel: Contribution to the B mode polarization.

CMB polarization is, however, a very promising tool for detecting a BGW background. The statistical properties of CMB linear polarization are fully characterized by two sets of spin-2 multipole moments with opposite parities [30]. As well known in the literature (see e.g. [31, 32]) the magnetic-type modes (B or curl modes) are produced by tensor metric perturbations and not scalar perturbations. A detection of B-mode polarization would thus provide good evidence for a primordial background of gravity waves.

As we can see from the bottom panel of Figure 5 the expected amplitude of the polarization B mode differs dramatically between the models. In particular, it is important to notice that a blue spectral index could enhance the B mode power at $\ell \sim 100$ by several orders of magnitude if compared with models with $n_T \leq 0$. Also in the plot we also report the expected sensitivity from the Planck experiment (dashed horizontal line). Most of the BGW spectra models are above the sensitivity of PLANCK.

We have simulated future cosmological data with a noise spectrum based on the expected Planck configuration. In particular we assume an experimental sensitivity ${w_p}^{-1/2} = 81\mu K$, beam size $\theta_{FWHM} = 7.1'$ and sky coverage $f_{sky} = 0.8$ We also assume a spatially flat model with parameters $h = 0.7$, $\omega_c = 0.120$, $\omega_b = 0.022$, $n_S = 1$ and $r = 0$. We find that the constraints on tensor modes expected for this model will be equally affected. The 2σ limit on r is indeed relaxed by $\sim 30\%$ from $r < 0.025$ to $r < 0.035$ when allowing for blue tensor spectra consistent with pulsar timing.

III. CONCLUSIONS

In this paper we have analyzed to which extent a primordial background of gravitational waves with positive spectral index is compatible with current observations. We have found that a considerable part of the parameter space of cosmological models is in agreement with current WMAP data and with upper limits coming from the LIGO experiment and from pulsar timing. Several non standard models can produce such a background and there is therefore no strong theoretical reason to exclude these models in current parameter estimation analysis. With this in mind, we have shown that, if the $n_T < 0$ assumption is relaxed, also parameters which are not directly related to the tensor component are affected. Especially, the scalar spectral index can be significantly bluer than in models which allow only for a red tilt of the tensor spectrum. Moreover, we have found a smaller but significant effect on the baryon and cold dark matter density constraints. This example shows once more that when assuming any limits on cosmological parameters it is very important to be aware of the model assumptions which went into their derivation.

Future Planck estimates can also be affected by BGW spectra, especially when constraints on r are considered. However they will be nearly entirely free of the additional degeneracies which are still seen in the WMAP data. As a final remark, we like to point out that blue spectra are able to produce significantly larger contribution to the B-mode polarization spectrum. An excess of B-mode polarization at $\ell \sim 100$ would therefore provide convincing evidence for the BGW models investigated here.

Acknowledgment
We thank Martin Kunz for discussions. RD is supported by the Swiss National Science Foundation. RC thanks Geneva University for hospitality. AM thanks the CERN Theory Division for financial support during the early stages of this work. This research was supported in part by the European Community's Research Training Networks under contracts MRTN-CT-2004-503369 and MRTN-CT-2006-035505. This research has been supported by ASI contract I/016/07/0 "COFIS".

[1] D. N. Spergel *et al.*, arXiv:astro-ph/0603449.
[2] L. Alabidi and D. H. Lyth, arXiv:astro-ph/0603539.
[3] H. Peiris and R. Easther, arXiv:astro-ph/0603587.
[4] D. Parkinson, P. Mukherjee and A. R. Liddle, arXiv:astro-ph/0605003.
[5] C. Pahud, A. R. Liddle, P. Mukherjee and D. Parkinson, arXiv:astro-ph/0605004.
[6] A. Lewis, arXiv:astro-ph/0603753.
[7] C. L. Reichardt *et al.*, arXiv:0801.1491 [astro-ph].
[8] U. Seljak, A. Slosar and P. McDonald, arXiv:astro-ph/0604335.
[9] J. Magueijo and R. D. Sorkin, arXiv:astro-ph/0604410.
[10] R. Easther and H. Peiris, arXiv:astro-ph/0604214.
[11] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D **74**, 023502 (2006) [arXiv:astro-ph/0605338].
[12] B. C. Friedman, A. Cooray and A. Melchiorri, Phys. Rev. D **74** (2006) 123509 [arXiv:astro-ph/0610220].
[13] F. Finelli, M. Rianna and N. Mandolesi, JCAP **0612** (2006) 006 [arXiv:astro-ph/0608277].
[14] L. F. Abbott and M. B. Wise, Nucl. Phys. B **244**, 541 (1984); A. A. Starobinsky, the JETP Lett. **30**, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. **30**, 719 (1979)]. V. A. Rubakov, M. V. Sazhin and A. V. Veryaskin, Unification Phys. Lett. B **115**, 189 (1982). R. Fabbri and M. d. Pollock, Phys. Lett. B **125**, 445 (1983).
[15] D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999) [arXiv:hep-ph/9807278].
[16] R. H. Brandenberger and C. Vafa, Nucl. Phys. B **316**, 391 (1989).
[17] A. Nayeri, R. H. Brandenberger and C. Vafa, Phys. Rev. Lett. **97**, 021302 (2006) [arXiv:hep-th/0511140].
[18] R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, Phys. Rev. Lett. **98**, 231302 (2007) [arXiv:hep-th/0604126].
[19] P. Creminelli, M. A. Luty, A. Nicolis and L. Senatore, JHEP **0612**, 080 (2006) [arXiv:hep-th/0606090]; E. I. Buchbinder, J. Khoury and B. A. Ovrut, Phys. Rev. D **76**, 123503 (2007) [arXiv:hep-th/0702154]; E. I. Buchbinder, J. Khoury and B. A. Ovrut, JHEP **0711**, 076 (2007) [arXiv:0706.3903 [hep-th]]; Creminelli and L. Senatore, JCAP **0711**, 010 (2007) [arXiv:hep-th/0702165].
[20] M. Baldi, F. Finelli and S. Matarrese, Phys. Rev. D **72**, 083504 (2005) [arXiv:astro-ph/0505552].
[21] N. Kaloper, L. Kofman, A. Linde and V. Mukhanov, JCAP **0610**, 006 (2006).
[22] R. Kallosh, J. U. Kang, A. Linde and V. Mukhanov, arXiv:0712.2040 [hep-th].
[23] A. Stewart and R. Brandenberger, arXiv:0711.4602 (2007).
[24] F. A. Jenet, G. B. Hobbs, K. J. Lee and R. N. Manchester, Astrophys. J. **625**, L123 (2005) [arXiv:astro-ph/0504458].
[25] F. A. Jenet *et al.*, Astrophys. J. **653**, 1571 (2006) [arXiv:astro-ph/0609013].
[26] B. Abbott *et al.* [LIGO Collaboration], Astrophys. J. **659**, 918 (2007) [arXiv:astro-ph/0608606].
[27] Hinshaw, G. *et al.* , Astrophys. J. Suppl. **170**, 288 (2007); Page, L. *et al.* , Astrophys. J. Suppl. **170**, 335 (2007).
[28] ESA, *PLANCK, the scientific programme*, 1, ESA-SCI(2005). arXiv:astro-ph/0604069.
[29] A. Lewis and S. Bridle, Phys. Rev. D **66**, 103511 (2002) (Available from `http://cosmologist.info`.)
[30] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55** (1997) 1830 [arXiv:astro-ph/9609170]; M. Kamionkowski, A. Kosowsky and A. Stebbins, Phys. Rev. D **55**, 7368 (1997) [arXiv:astro-ph/9611125].
[31] M. Kamionkowski, A. Kosowsky and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997) [arXiv:astro-ph/9609132]; U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997) [arXiv:astro-ph/9609169].
[32] L. Pagano, A. Cooray, A. Melchiorri and M. Kamionkowski, arXiv:0707.2560 [astro-ph].